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Ship dynamics identification using simulator and sea trial data

Philip S.E. Farrell

Defence R&D Canada – Toronto
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Abstract

The training effectiveness of the Maritime Surface and Subsurface (MARS) Virtual Reality Simulator (VRS) was determined by means of instructor ratings. The purpose of this report is to identify the dynamics of the virtual and real ships so that objective measures of ship handling ability could be derived. Determining the transfer of training requires knowledge of the ship dynamics. This paper identifies the ship dynamics for a CF Bay-Class ship as well as a simulated ship. Simulation and sea trial data are collected and used to identify parameter values for the models. Sources of error came from the sea state, crew behaviours, and the Differential Global Positioning System. Despite the sources of error, the data were relatively *clean* and the identification exercise was able to proceed with minimal post processing. The analysis produced a piecewise linear and continuous model description for the ship's dynamics, with numerical coefficients. The advantage of the model is that it is computationally simpler than the full six degree of freedom model, and still captures over 90% of the experimental variance. The algorithms developed in this paper may be implemented to generate a real time ship dynamics model that could be downloaded into a portable MARS VRS system for onboard training and rehearsal.

Résumé

L'efficacité de la formation du simulateur de réalité virtuelle (SRV) maritime de surface et de subsurface (MARS) a été déterminée à l'aide de cotes d'instructeurs. Le but du présent rapport est de déterminer la nature de la dynamique des navires virtuels et réels de manière à ce que des mesures objectives de la capacité de manipulation des navires soient dérivées. La détermination du transfert de formation nécessite des connaissances sur la dynamique du navire. Le présent document précise la dynamique du navire pour un navire de classe Bay des FC, ainsi que pour un navire simulé. Les données de simulation et d'essais en mer sont regroupées et utilisées pour déterminer les valeurs des paramètres des modèles. Les sources d'erreurs proviennent de l'état de la mer, du comportement des équipages et du système de positionnement global différentiel. En dépit des sources d'erreurs, les données ont été relativement *nettes* et l'exercice d'identification a permis de n'avoir à effectuer qu'un traitement ultérieur minimal. L'analyse a produit une description de modèle linéaire et continue par morceaux pour la dynamique du navire, avec des coefficients numériques. L'avantage du modèle est sa plus grande simplicité sur le plan du calcul en comparaison du modèle à 6 degrés de liberté et saisit toujours 90 % de l'écart expérimental. Les algorithmes élaborés dans le présent document pourraient être mis en oeuvre en vue de générer un modèle de dynamique de navire en temps réel qui pourrait être téléchargé dans un système portatif MARS VRS pour la formation à bord et les répétitions.

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Executive summary

The training effectiveness of the Maritime Surface and Subsurface (MARS) Virtual Reality Simulator (VRS) was determined by means of instructor ratings. The purpose of this report is to identify the dynamics of the virtual and real ships so that objective measures of ship handling ability could be derived. This paper identifies the ship dynamics for a CF Bay-Class ship as well as a simulated ship. Simulation and sea trial data are collected and used for a system's identification exercise. The analysis produced a piecewise linear and continuous model that included numerical coefficients.

The advantage of the model is that it is simpler than the full six degree of freedom model, and still captures over 90% of the actual ship dynamics. The algorithms developed in this paper may be implemented to generate a real time ship dynamics model that could be downloaded into a portable MARS VRS system for onboard training and rehearsal.

Sommaire

L'efficacité de la formation du simulateur de réalité virtuelle (SRV) maritime de surface et de subsurface (MARS) a été déterminée à l'aide de cotes d'instructeurs. Le but du présent rapport est de déterminer la nature de la dynamique des navires virtuels et réels de manière à ce que des mesures objectives de la capacité de manipulation des navires soient dérivées. Le présent document précise la dynamique du navire pour un navire de classe Bay des FC, ainsi que pour un navire simulé et des données d'essais en mer sont regroupées et utilisées en vue d'un exercice d'identification de système. L'analyse a produit une description de modèle linéaire et continue par morceaux pour la dynamique du navire, avec des coefficients numériques.

L'avantage du modèle est sa plus grande simplicité sur le plan du calcul en comparaison du modèle à 6 degrés de liberté et il saisit toujours 90 % de la dynamique réelle du navire. Les algorithmes élaborés dans le présent document pourraient être mis en oeuvre en vue de générer un modèle de dynamique de navire en temps réel qui pourrait être téléchargé dans un système portatif MARS VRS pour la formation à bord et les répétitions.

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Introduction

The Canadian Forces desire effective yet inexpensive methods for training student Officers Of the Watch (OOW) (Magee, 1997). To that end, DCIEM has developed the Maritime Surface and Subsurface (MARS) Virtual Reality Simulator (VRS), which includes voice recognition, 2D and 3D computer graphics, expert systems, and precision tracking. A series of simulator and sea trials, performed in the early 90's, showed that the MARS VRS afforded transfer of training (Magee, 1997). Instructors' ratings were used to show that students learned in the simulator; their simulator results partially predicted sea performance, and the simulator afforded positive transfer of training.

One feature of the MARS VRS is the ability to generate and record path data (longitude and latitude positions over time) while the student trains in the simulator. These data can be used to calculate an objective score – the average difference between the student ship's path and an ideal path over a manoeuvre (Farrell, 2002). The smaller the score, the better the performance.

Bay-class ships were retrofitted with a Differential Global Positioning System (DGPS) to collect path data at sea. Identical manoeuvres were performed in the simulator and objective scores were calculated for students in the simulator and then at sea, as well as students who trained at sea only. The objective scores were applied to the same analyses as the subjective scores from the instructors' ratings (Magee, 1997). Namely:

1) *Learning*: simulator students showed a performance improvement over successive trials for both subjective and objective techniques.

2) *Performance Prediction*: Subjective scores showed that simulator performance partially predicts performance at sea while objective scores yielded no significant findings.

3) *Transfer of Training*: Both subjective and objective scores showed that simulator students at sea out performed the control group on average, thus indicating transfer of training.

The objective scoring method is reliable, repeatable, and can measure learning and the transfer of training just as effectively as the subjective scoring method. One shortcoming is that the objective method is based solely on performance, while the subjective scores also captures the ability to plan, demonstrate bridgemanship, and operate safely. The objective method can be used along side the instructor's ratings.

Before performing these analyses, the ship dynamic model had to be identified in order to derive the objective score. The derivation began with the OOW-ship interaction model in Figure 1, which is based on Perceptual Control Theory (PCT; Powers, 1973). A relationship between the OOW decision-making function and the path data was found from the model (Farrell, 2002) given that:

- The OOW has internalised a desired path that is identical to the "text book" solution.
- The perceived and actual paths are equal, therefore the perception function is 1.
- <u>The ship dynamic model is identified</u>.

This paper focuses on identifying a ship dynamics model using simulator and sea trial data.

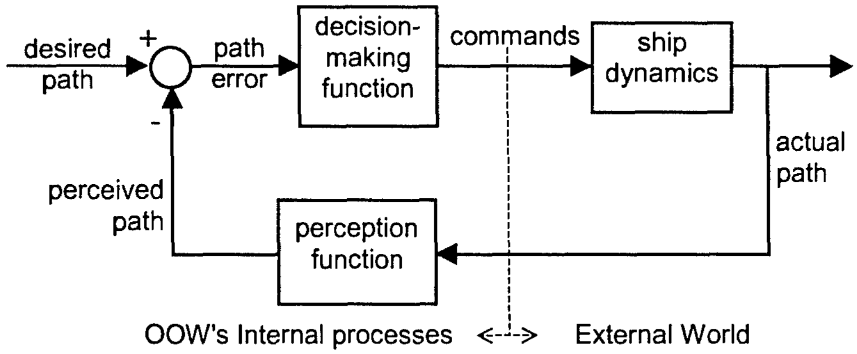

Figure 1 *PCT Model of Ship Manoeuvres under the Control of the OOW*

Ship dynamic models can be categorized as generic and specific models. Generic models, such as (Mandel, 1969) and the MARS VRS model (based on Gong, 1993), are high fidelity, non-linear, six degree of freedom, coupled equations that cover the full range of ship dynamics. However, finding numerical values for the model is often challenging because they are not readily available. Also, these models are computationally intensive, although advances in computer technology make real time complex calculations affordable.

Specific models are well suited for real time simulation. These models are usually linear, zero- or first-order differential equations. The fidelity is necessarily lower than the generic models. However, parameters are identified that best fit a specific range of ship dynamics for a given sea state, payload, and manoeuvre. For example, a specific model may fit exactly a port 15 turning circle, but may turn slightly inside a port 35 circle and slightly outside a port 7 circle for the same parameter set. Theoretically, a library of parameter sets that fit every manoeuvre and condition could be generated.

A hybrid model (part specific and part generic) is proposed that consists of linear first order differential equations with non-linear terms derived from conservation of momentum and energy equations. The model potentially reduces the number of parameter sets it would take to model the full range of manoeuvres and conditions. With the hybrid model, one can:

- Compare the MARS VRS software model with the hybrid model,
- Compare the MARS VRS and sea trial data via the hybrid model.
- Note the advantages of the hybrid model for the OOW at sea.

The identification of the hybrid model begins by noting that ship speed data look like exponential responses to a target speed input. These graphs can be fitted with a first order linear differential equation. The target values, however, seem to depend on the commanded speed and rudder position, and this relationship is expressed with non-linear gains. These gains are derived from first principles, and simulator and sea path data are used to find numerical values for their parameters.

The path data are generated from ships that perform specific, well-defined manoeuvres. Typically for parameter identification exercises, a wide range of sinusoidal signals (frequency and amplitudes) are used to excite the system, and the results may be used in frequency analyses (Van de Vegte, 1986), from which the fundamental characteristics of the ship dynamics can be deduced. However, ship manoeuvres are restricted to standard manoeuvres: acceleration profiles (speed step input), and turning circles (rudder step input). Although the identification technique is less robust using step inputs than sinusoidal inputs, the resultant model sufficiently describes the range of manoeuvres that the ships normally perform.

Ship dynamic models are identified for both the MARS VRS and Bay-class ships at sea that differ only by the parameter values. This report presents the model in general terms, outlines the data collection activity, and uses the data to identify the parameter values for the two models.

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Ship Dynamics Model

The OOW-ship interaction model in Figure 1 is recast into standard Classical Control Theory (CCT) terms as shown in Figure 2. The signal and function definitions are listed below:

r	– system reference	e	– error signal	G_c	– controller transfer function
c	– system output	u,α	– command inputs	G	– plant transfer function
				H	– feedback transfer function

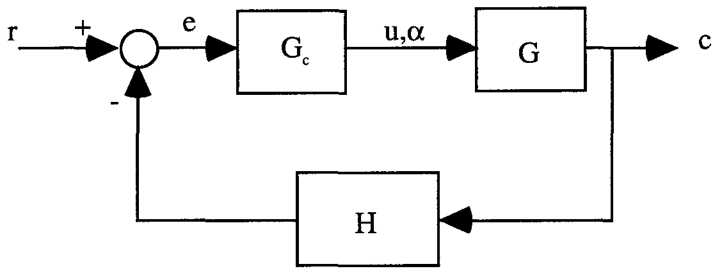

Figure 2 *Standard Feedback Control System*

In this context, G is the ship dynamics model to be identified. u and α are speed and rudder commands, and c is the ship's path. The structure of G is derived from dynamics and kinematics relationships for ship motion. The numerical parameters of G are found given the model inputs (commands) and output (path data).

Relationship between Path and Velocity

Starting from the system response and working backwards, the relationship between the ship's path and velocities is given below. The ship's path in Cartesian co-ordinates (x,y) is:

$$\begin{aligned} x(t) &= d(t)\cos\{\theta(t)\} \\ y(t) &= d(t)\sin\{\theta(t)\} \end{aligned} \tag{1}$$

The distance travelled (d) along a curvilinear path and heading (θ) are found by integrating the tangential velocity (v) and the angular velocity (ω) as follows (Meriam, 1980):

$$\begin{aligned} d(t) &= \int v(t)dt \\ \theta(t) &= \int \omega(t)dt \quad . \end{aligned} \tag{2}$$

Transforming equation (2) into the Laplace Domain allows algebraic operations to be performed with the integrals and derivatives (Van de Vegte, 1986):

$$\begin{aligned} d(s) &= L\{d(t)\} = L\left\{\int v(t)dt\right\} = \frac{v(s)}{s} \\ \theta(s) &= L\{\theta(t)\} = L\left\{\int \omega(t)dt\right\} = \frac{\omega(s)}{s} \quad . \end{aligned} \tag{3}$$

s is the transformation variable and the operator, L{ } transforms the integral from the time domain to the Laplace domain. The block diagram in Figure 3 shows the ship's path as a function of its velocities.

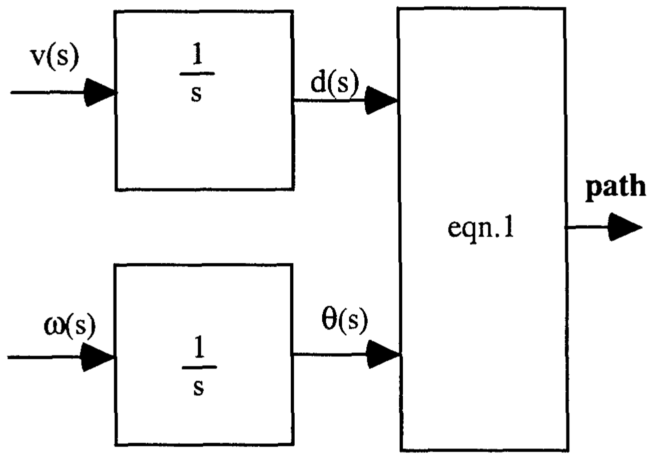

***Figure 3** Relationship between Path and Tangential and Angular Velocities*

Relationship between Velocity and Command

The ship's tangential and angular velocities do not instantly reach their target values on command. The data show an initial transient response. The functions G1(s) and G2(s) in Figure 4 represent the transient dynamics between the actual velocities, v(s) and ω(s), and their target velocities, $v_t(s)$ and $\omega_t(s)$.

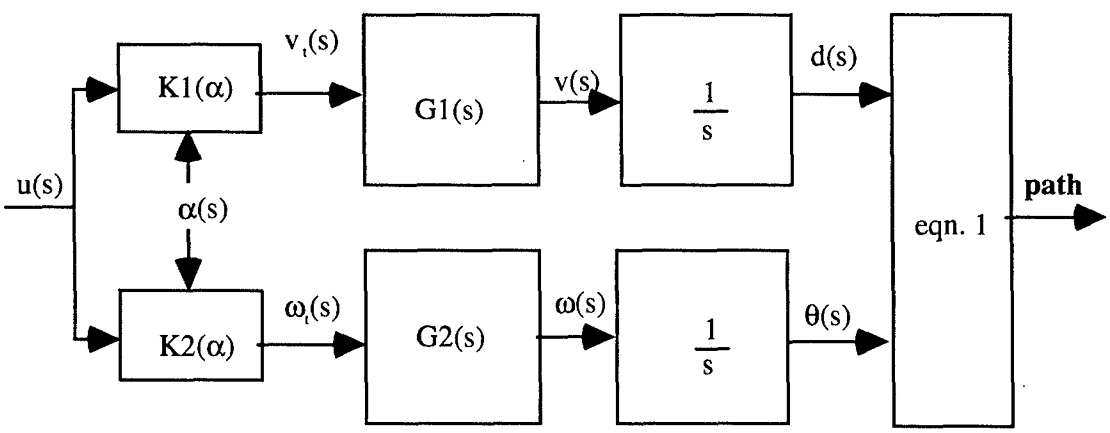

***Figure 4** Ship's Dynamic Model in Block Diagram Form*

Working backwards still, the data show non-linear relationships between v_t and ω_t, and the speed and rudder commands, u and α, respectively. The non-linear gains, K1(α) and K2(α), represent a drop in tangential and angular velocities due to increasing drag during turning circles. Figure 4 shows the relationship between the OOW's commands and the ship's path. The next step is to collect experimental data, and then use them to determine the explicit forms of the non-linear gains and the transfer functions, and their parameter values.

Data Collection

Path data from simulation and sea trials were used to identify the Bay-class ship dynamics. The trial's objective was to excite the ship with well-defined step inputs and record the response. Historically, turning circles are standard manoeuvres used to define a specific ship's characteristics. The turning circles and acceleration profiles found during this exercise may be used to update the thirty-year-old performance data for the Bay-class ships in the CF.

Sources of error for data collection include 1) crew behaviour, 2) the sea state, and 3) DGPS.

1. The bridge and engine room crews interpret and convert the OOW's rudder and speed commands into a rudder position and a number of engine revolutions per minute. However the crew at sea make additional judgements based on the current sea state, payload, and operational requirements. This behaviour was not measured nor controlled for during the sea trial data. In contrast, the simulated crew's behaviour is repeatable, and so any error produced by the simulated crew would be repeatable as well.

2. Similarly, the sea state for the simulated ship is the same for all manoeuvres. However, at sea, changing winds, water currents, wake interference, etc., disturb the ship's path. The data show that these disturbances are the largest source of error.

3. The DGPS generates longitude/latitude positions as the ship moves through the water. Its resolution was two to three feet. Occasionally, one of several satellites would move out of position and the positioning accuracy could not be guaranteed. A few runs needed to be manually adjusted for the DGPS induced offset.

Table 1 provides a list of standard speed and rudder manoeuvres performed during the trials. The first row represents the acceleration profiles, where the ship accelerated from 0 knots to the target speed and then decelerated back to 0 knots. The next rows represent port and starboard turning circles at three rudder positions, 7, 15, and 35 degrees, and three speeds, 9, 12, and 15 knots. Each manoeuvre was given a priority number (bracketed number) if time ran out.

Table 1. *Required Standard Manoeuvres*

	TARGET SPEED				
RUDDER ANGLE	***9 kts***	***10 kts***	***12 kts***	***14 kts***	***15 kts***
0°	0-9 9-0 (1)	0-10 10-0 (4)	0-12 12-0 (2)	0-14 14-0 (5)	0-15 15-0 (3)
7°	port/stbd (6)		port/stbd (9)		port/stbd (12)
15°	port/stbd (7)		port/stbd (10)		port/stbd (13)
35°	port/stbd (8)		port/stbd (11)		port/stbd (14)

A computer recorded and time stamped the ship's position and the OOW's commands every five seconds in the simulator, and every second at sea. At sea, a crew member logged the command and its time of issue, and a tape cassette recorded critical parts of the bridge conversation such as the start and end times of each manoeuvre. These recordings were used to parse the raw data into the manoeuvres defined by Table 1. Annex A provides a detailed list of the manoeuvres and the times that they were performed at sea.

```
                              ⋮
#001,Feb25,155024,4826.3660N,12326.8205W,   0,D2,  6,  1N,350T,14,20,25,29
#001,Feb25,155025,4826.3657N,12326.8206W,   0,D2,  6,  1N,350T,14,20,25,29
#001,Feb25,155026,4826.3654N,12326.8207W,   0,D2,  6,  1N,351T,14,20,25,29
#001,Feb25,155027,4826.3652N,12326.8208W,   0,D2,  6,  1N,350T,14,20,25,29
#001,Feb25,155028,4826.3648N,12326.8210W,   0,D2,  6,  1N,349T,14,20,25,29
#001,Feb25,155029,4826.3645N,12326.8214W,   0,D2,  6,  1N,349T,14,20,25,29
#001,Feb25,155030,4826.3642N,12326.8218W,   0,D2,  6,  1N,349T,14,20,25,29
                              ⋮
```

Figure 5 *A sample of the DGPS data file.*

Figure 5 shows a portion of a raw DGPS data file, from which the ship's path, speed, heading, and angular velocity are derived for each manoeuvre. The third (time), fourth (latitude), and fifth (longitude) columns were used to generate heading, speed and other derived data. Annex B contains the equations for converting the raw data into the derived data.

Sea trials were performed with HMCS Thunder on 25 February 1994 in Juan de Fuca Strait, off the coast of Esquimalt, British Columbia. The wind was NNW at 25 knots with negligible tidal stream. The manoeuvres were performed from about 1100 to 1500 hours local time. All manoeuvres listed in Table 1 were performed except for those with the rudder angle set to 35 degrees. Thirty degrees was the maximum acceptable angle for the given sea state conditions on that day. Figure 6 shows the complete path the ship took on 25 February 1994 in the Strait of Juan Du Fuca.

Annex C includes the acceleration profiles and plots of the path of the ship during the manoeuvre. Note that the ship never achieved a speed of zero knots due to wind, waves, and ship momentum. A standard presentation form for turning circles are archived in Annex D. The tables list the amount of turn degrees, the advance and transfer yards, the speed at every 15 degrees, the average turning rate, and the time for each quarter turn. The inset figure shows the path of the ship in the water during the manoeuvre.

Overall, the sea trial data were relatively *clean* despite the sources of error. Tighter control on the start and end of manoeuvres would have been desirable. For example, to terminate a turning circle, the OOW gave a heading command, not a rudder command of zero degrees. In the former case, the crew might give opposite rudder in order to quickly reduce the ship's angular momentum – although the goal was to see how the ship behaved when decelerating at a single rudder position. The wind and currents were not a significant factor, however, the turning circle data clearly show a second order oscillation on top of the acceleration profiles due to wake interference. As expected, the DGPS data are noisy as seen in the path, heading, and speed plots. Recent advances in DGPS technology will reduce the noise significantly for future data collection exercises.

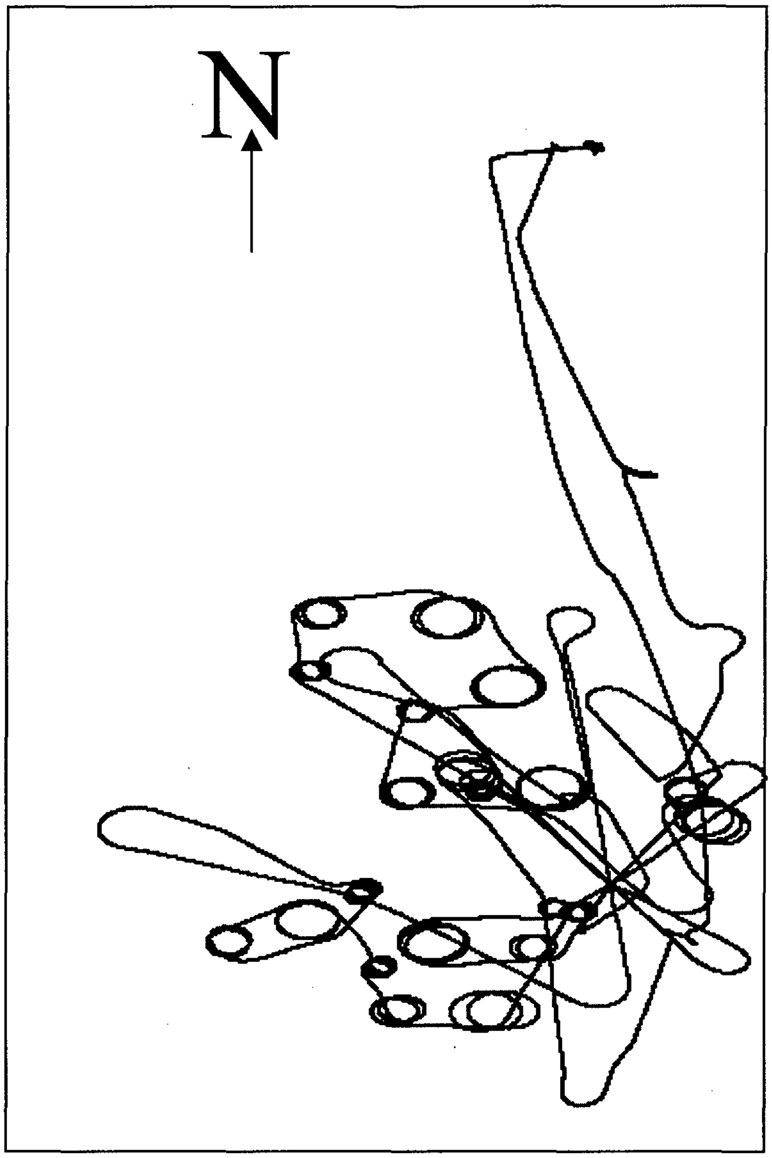

***Figure 6** HMCS Thunder on 25 Feb. 94 in the Strait of Juan Du Fuca*

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Transfer Function Identification

Table 1 provides the rudder and speed commands (model inputs) and Annexes C and D contain the ship's path (model output). The transfer functions and non-linear gains in Figure 4 now can be identified from the model inputs and output.

Identifying G1(s)

The transient dynamics of the tangential velocity, G1, is the first of four functions to be identified. The analysis begins by obtaining the linear speed data (special case of tangential velocity). The ship accelerated from an initial low speed to the command speed, maintained a steady state speed for several minutes, and then decelerated back to the low speed value, thus producing velocity profiles. The low speed value was zero and about 2 knots for the simulation and sea trials, respectively. The commanded speeds were 9, 10, 12, 14 and 15 knots. The data collected during the 0-10 and 10-0 knots sea trial manoeuvres were not used since the data included unexpected speed command changes.

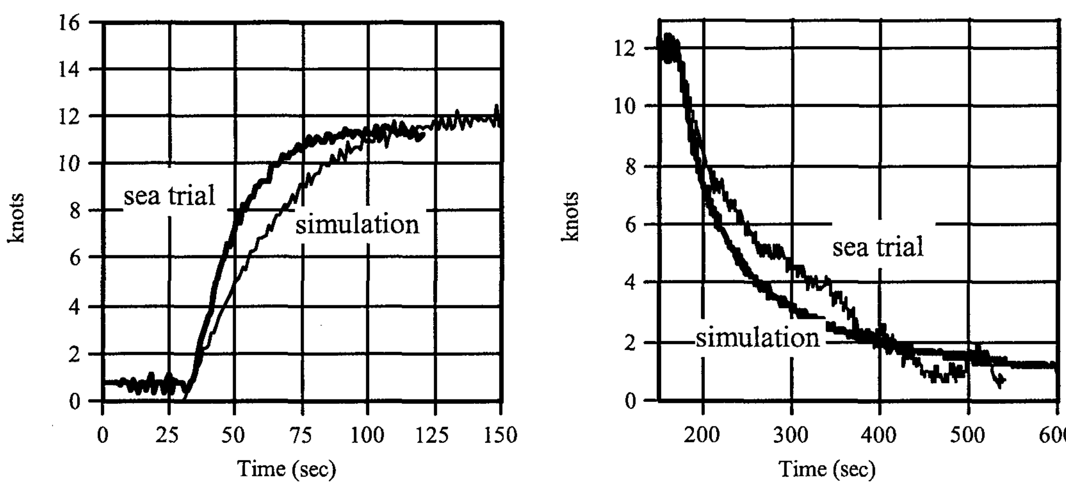

***Figure 7** Linear Speed Data for 0-12 Knots and 12-0 Knots*

Figure 7 is a sample plot of a 12 knots linear speed command for both simulation and field trial data. Curve fitting the raw data effectively filters the data, and the curve fit is used to find the model's parameter values. However, as a final check, the model is correlated against the raw data – not the curve fit.

Based on exponential-type responses, several models for G1 were proposed including ideal and quadratic lags, which can reproduce similar responses. However, a simple lag model yielded the best data correlation. A simple lag model also produces an exponential response, and it is expressed in the Laplace domain as follows:

$$\frac{v(s)}{v_t(s)} = G1(s) = \frac{a}{s+b} \tag{4}$$

a and b are parameters related to the model's gain and time constant. The steady state speed, v_{ss} turns out to be a fraction of v_t (which agrees with the experimental observations) as follows:

$$\begin{aligned} v_{ss} &= \lim_{s \to 0} sv(s) \text{ (Final Value Theorem : Van de Vegte, 1986)} \\ &= \lim_{s \to 0} s \frac{v_t}{s} \frac{a}{s+b} \\ &= \frac{a}{b} v_t \end{aligned} \tag{5}$$

The parameters a and b are found by first setting $\alpha = 0$, $K1(\alpha)$, = 1 and $K2(\alpha) = 0$ such that the target velocity is equal to the commanded speed, $v_t(s) = u(s)$. The simple lag parameters are not the same for the acceleration and deceleration manoeuvres. That is, the model takes on a variable structure (Farrell, 1992) where the model parameters (a and b) are different for each manoeuvre. This model is strictly non-linear, however, each manoeuvre is linear and can be analysed separately. And so, this is sometimes called a "piecewise linear model".

Consider the time domain expression of equation 4 and substitute two data points as follows:

$$\begin{aligned} \dot{v}(t_1) + bv(t_1) &= av_t \\ \dot{v}(t_2) + bv(t_2) &= av_t \end{aligned} \tag{6}$$

Equation 6 contains two equations with two unknowns, a and b. v(t) comes from fitting a fifth order polynomial to the raw path data, and differentiating the polynomial. $\dot{v}(t)$ is found by differentiating the polynomial again. Two time points were selected and substituted into equation 6 to solve for the model parameters. Then the path data was generated for these new parameters. The correlation between the model's path data and the raw data was calculated. This procedure was repeated with different sets of times, until the correlation was greater than 0.90 (arbitrarily chosen). Annex E describes the parameter evaluation procedure in detail, along with the computer routines.

Table 2. G1(s) *Model Parameter Results from Simulation Data*

	MARS VRS TRIALS					
	acceleration: 0 to v_t			***deceleration: v_t to 0***		
v_t (knots)	***a***	***b***	***r***	***a***	***b***	***r***
9	0.021	0.022	0.98	0.011	0.011	0.99
10	0.023	0.022	1.00	0.013	0.013	0.98
12	0.028	0.024	0.99	0.008	0.008	0.88**
14	0.030	0.028	0.99	0.012*	0.012*	0.99*
average	0.025	0.024	0.99	0.012	0.012	0.98

*These values do not satisfy equation 5 but they do produce a good fit.

**These parameters are not used in the average since the correlation is less than 0.90.

Tables 2 and 3 summarise a and b values for simulation and sea trial data, respectively. Separate model parameters were found for acceleration (power) and deceleration (no power) manoeuvres. The target velocities ranged from 9 to 14 knots. r is the correlation derived in Annex F. Note that the deceleration parameters are similar for both sea and simulation trials, while the acceleration results differ by a factor of 2.6. The average values from Tables 2 and 3 are substituted into the model for G1(s) from equation 4 and presented in Table 4.

Table 3. G1(s) *Model Parameter Results from Sea Trial Data*

	SEA TRIALS					
	acceleration: 0 to v_t			***deceleration:*** v_t ***to 0***		
v_t (knots)	***a***	***b***	***r***	***a***	***b***	***r***
9	0.055	0.045	0.98	0.009	0.012	0.90
12	0.070	0.067	0.98	0.013	0.014	0.89**
14	0.078*	0.070*	0.98*	0.013	0.013	0.97
average	0.068	0.061	0.98	0.011	0.013	0.94

*These values do not satisfy equation 5 but they do produce a good fit.

**These parameters are not used in the average since the correlation is less than 0.90.

Table 4. *Expressions for* G1(s)

SIMULATION		**SEA TRIAL**	
acceleration	***deceleration***	***acceleration***	***deceleration***
$\frac{0.025}{s+0.024}$	$\frac{0.012}{s+0.012}$	$\frac{0.068}{s+0.061}$	$\frac{0.011}{s+0.013}$
r = 0.99	r = 0.98	r = 0.98	r = 0.94

Identifying G2(s)

The angular velocity dynamics, G2, is the second function to be identified using turning circle data. A typical manoeuvre begins just after the ship reaches a constant linear speed and heading, say 12 knots and bearing 090 (Figure 8). The rudder turns immediately to a predetermined position, say port 15. The transient response is about 20 seconds long followed by a steady state response of two to three circles at a constant angular velocity.

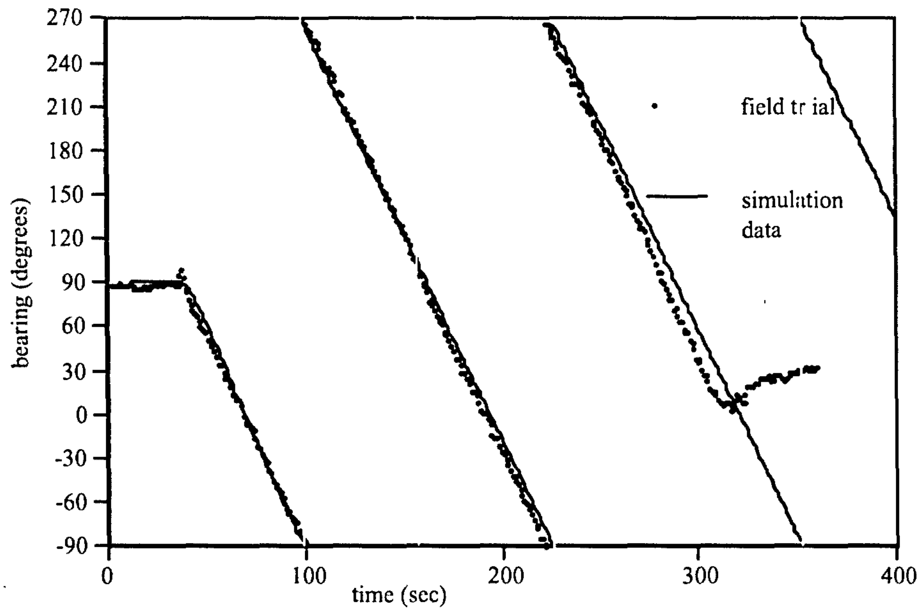

Figure 8 *Sea trial and simulation data showing the ship's heading versus time for a 12 knot, port 15 turning circle manoeuvre.*

Experimental data show that the steady state and target angular velocities are equal. Therefore, an ideal lag model (i.e., a = b) was used for G2(s) as follows:

$$\frac{\omega(s)}{\omega_t(s)} = G2(s) = \frac{c}{s+c} \tag{7}$$

c is related to the model's time constant and ω_t is the target angular velocity. Using the inverse Laplace transform, the analyst can find an expression for $\omega(t)$ and integrate it to find the heading (θ) as follows:

$$\begin{aligned} \omega(t) &= \omega_t\left(1-e^{-ct}\right) \\ \theta(t) &= \theta_o + \omega_t t - \frac{\omega_t}{c}\left(1-e^{-ct}\right) \\ \theta(t) &= \theta_o + \omega_t t - \frac{\omega_t}{c} \ldots t \to \infty \end{aligned} \tag{8}$$

θ_o, and ω_t are found directly from plots similar to Figure 8, where θ_O is the initial heading and ω_t represents the slope of the line. c is calculated by selecting a point on the line and solving the steady state expression in equation 8.

The parameters ω_t and c were solved from simulation and sea trial data and are summarised in Tables 5 and 6, respectively. The parameters are categorised in terms of the circle direction: (port and starboard) and manoeuvre commands (speed and rudder).

Table 5. G2(s) *Parameter Results from Simulation Data*

	MARS VRS TRIALS			
	Port		***Starboard***	
Manoeuvre	ω_t	c	ω_t	c
9 kts 7°	-1.30	0.21	1.29	0.22
9 kts 15°	-2.26	0.34	2.13	0.37
9 kts 30°	-3.18	0.24	2.62	0.26
12 kts 7°	-1.73	0.24	1.72	0.22
12 kts 15°	-3.14	0.22	2.83	0.23
12 kts 30°	-4.34	0.21	3.49	0.22
15 kts 7°	-2.17	0.26	2.15	0.28
15 kts 15°	-3.92	0.21	3.54	0.32
15 kts 30°	-5.34	0.22	4.36	0.21
Average		**0.24**		**0.26**

Table 6. G2(s) *Parameter Results from Sea Trial Data*

	SEA TRIALS			
	Port		***Starboard***	
Manoeuvre	ω_t	c	ω_t	c
9 kts 7°	-1.49	0.43	1.66	0.03
9 kts 15°	-2.21	0.39	2.28	0.26
9 kts 30°	-2.87	0.56	3.00	0.37
12 kts 7°	-2.06	0.25	2.33	0.07
12 kts 15°	-2.84	0.24	4.26	0.17
12 kts 30°	-3.70	0.55	4.11	0.16
15 kts 7°	-2.47	0.11	2.73	0.16
15 kts 15°	-3.60	0.11	3.76	0.14
15 kts 30°	-5.44	0.19	5.14	0.13
Average		**0.31**		**0.17**

The sea trial parameter results vary widely, likely due to wake interference. Wake interference is not modelled in the simulator. However, the average port and starboard time constants for the sea trial is 0.24, which is similar to the simulation average of 0.25. Substituting these average values into equation 7 yields the model of *G2(s)* as given in Table 7.

Table 7 *Expressions for* G2(s)

SIMPLE LAG MODEL FOR *G2(s)*	
Simulation	***Sea Trial***
$\frac{0.25}{s + 0.25}$	$\frac{0.24}{s + 0.24}$

Identifying K1(α)

K1(α) is the third function to be identified. This non-linear gain acts to diminish the tangential speed during a turning manoeuvre. Figure 9 shows the speed versus time curves for 9, 12, and 15 knots, and for a rudder angle of port 15. Here, the tangential speed drops to 0.7 of the commanded speed. The complete set of tangential speed graphs with the corresponding attenuation factor for all speed and rudder angle combinations are contained in Annex G.

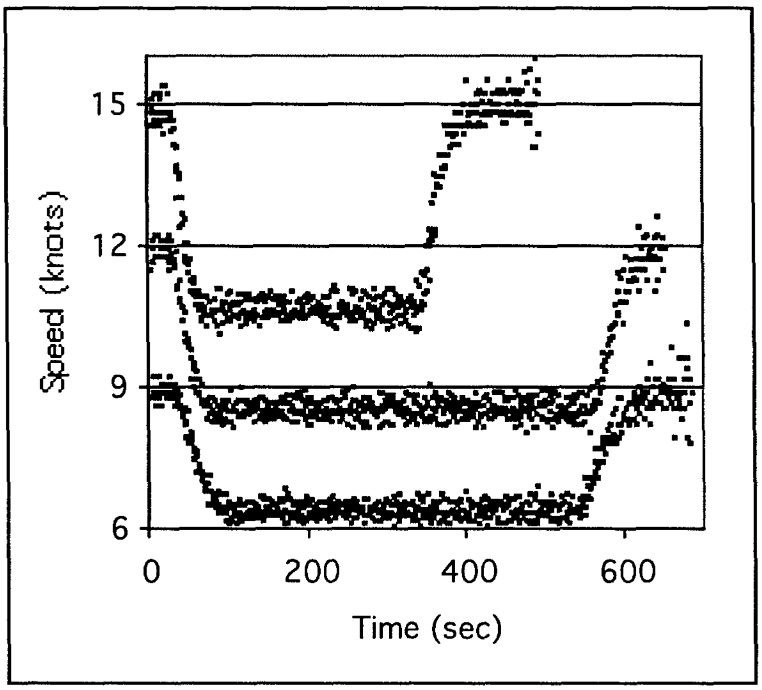

Figure 9 *Loss of Tangential Velocity for Simulation Port 15:* $v_{ss} = 0.71 v_{ss\alpha}$.

To account for this drop in speed, the forces on the ship's hull are examined from first principles. The ship's hull is represented by an airfoil moving through a fluid. The thrust is constant since the engine speed (rpm's) does not change throughout the turning circle manoeuvre. The drag is equal but opposite to the thrust, and it is constant during the steady state portions of the manoeuvre, and the drag force is expressed as follows (White, 1979):

$$D = \tfrac{1}{2} C_d v_{ss\alpha}^2 \rho l h \quad , \tag{9}$$

where:

- D – drag force
- C_d – drag coefficient
- $v_{ss\alpha}$ – steady state speed during a turning manoeuvre
- ρ – water density
- l – ship's length
- h – hull's extent into the water.

For a constant drag force, the drag coefficient must increase as the speed decreases, and visa-versa. The expression for the drag coefficient as a function of the rudder angle, α, is given as follows (White, 1979):

$$C_d = C_{do} + \frac{4\pi \, l/h \sin^2\alpha}{(l/h + 2)^2} \tag{10}$$

C_{do} is the drag coefficient when $\alpha = 0$ (at the beginning of the manoeuvre). Energy is conserved before and during the turning circle. Since the thrust is constant throughout, the drag force before and during the turning circle be equated as follows:

$$\begin{aligned} D_o &= D \\ C_{do} v_{ss}^2 &= C_d v_{ss\alpha}^2 \\ v_{ss}^2 &= \left(1 + \frac{4\pi \, l/h \sin^2\alpha}{C_{do} (l/h + 2)^2}\right) v_{ss\alpha}^2 \\ v_{ss\alpha} &= v_{ss} \sqrt{\frac{1}{1 + k_1 \sin^2\alpha}} \end{aligned} \tag{11}$$

k_1 is a constant parameter that includes C_{do}, l, and h. Equation 11 infers that the steady state speed for a given rudder position is some fraction of the zero rudder angle steady state speed. Therefore, the non-linear gain, $K1(\alpha)$ in Figure 4, is the radical in equation 11 that causes a reduction in the steady state speed during turning circles as follows:

$$K1(\alpha) = \sqrt{\frac{1}{1 + k_1 \sin^2\alpha}} \tag{12}$$

The ratio between the steady state speeds is reported on each graph in Annex G, and summarised in Table 8. The value of k_1 is found by fitting the results in Table 8 with equation 12, as shown in Figure 10. $k_1 = 14.5$ and $k_1 = 4.4$ for the simulation and sea trials, respectively. These values are different most likely due to the wake interference source of error. In summary, the non-linear gain has been identified which accounts for the loss of speed during a turning circle manoeuvre. The expressions of K1(α) are given in Table 9.

***Table 8** Steady State Gains*

$K1(\alpha) = V_{ss\alpha}/V_{ss}$				
	Simulation		***Field Trial***	
Rudder Angle	***Port***	***Stbd***	***Port***	***Stbd***
0°	1.00	1.00	1.00	1.00
7°	0.90	0.90	0.91	0.95
15°	0.71	0.71	0.84	0.89
30°	0.49	0.50	0.72	0.75

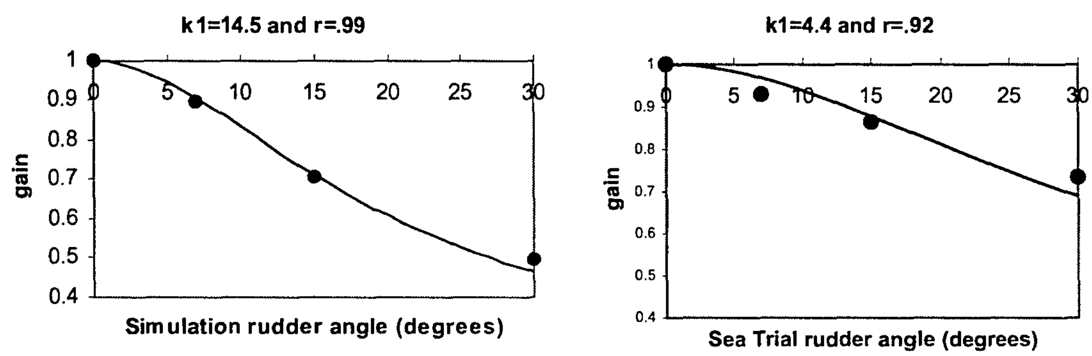

***Figure 10** Gain versus Rudder Angle (Equation 12) fitted to simulator and sea trial data (Table 8)*

***Table 9** Expressions for K1(α)*

K1(α)	
Simulation	***Sea Trial***
$\sqrt{\dfrac{1}{1+14.5\sin^2\alpha}}$	$\sqrt{\dfrac{1}{1+4.4\sin^2\alpha}}$

Identifying K2(α)

The final function to be identified is the non-linear gain, K2, which acts to lower the angular velocity during a turning manoeuvre. The data show that the steady state angular velocity varies with the rudder angle as well the speed as shown in Figure 11 (simulation data only).

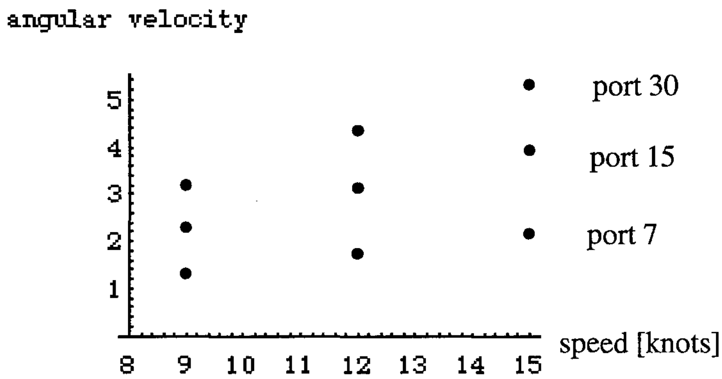

Figure 11 *Angular velocity steady state response as a function of speed and rudder angle*

The plots in Figure 11 form straight lines, where the slopes for port 7, 15, and 30 are 0.14, 0.26, and 0.36, respectively. When lines extend backwards, they intersect the origin; that is, the angular velocity is zero when the rudder position is zero. From the graph, $\omega_t = K2(\alpha)u$, where K2(α) is the slope of the lines. Also, from kinematics, $u = r(\alpha)\omega_t$, where $r(\alpha)$ is the instantaneous radius of curvature (Meriam, 1980). Therefore, K2(α) is the inverse of the instantaneous radius of curvature.

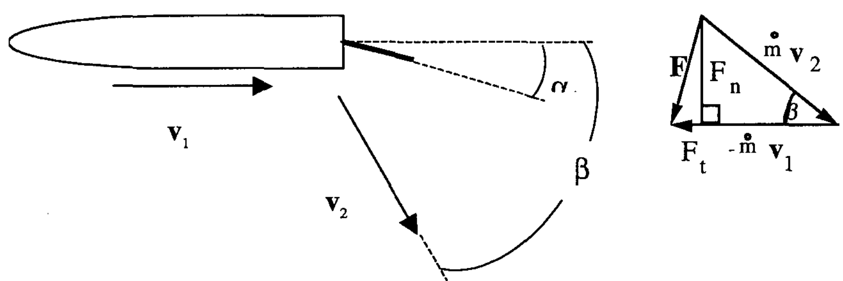

Figure 12 *Ship Icon Depicting Water Flow Redirection*

To find the form of K2(α), consider an ideal ship and rudder moving through water with negligible resistance. The rudder redirects the water flow thus changing the system momentum by changing the water flow direction as shown in Figure 12. The force due to the change in momentum is expressed as follows:

$$F = \dot{m}(\mathbf{v}_2 - \mathbf{v}_1) \tag{13}$$

where $\dot{m}$ is the mass flow rate and $\mathbf{v}_1$ and $\mathbf{v}_2$ are flow speeds. Given that $|\mathbf{v}_1| = |\mathbf{v}_2| = u$ and $\dot{m} = \rho u A$, the tangential and normal components of the force are as follows:

$$F_t = u^2\rho A(1-\cos(\beta))$$
$$F_n = u^2\rho A\sin(\beta) \tag{14}$$

ρ is the water density and A is the mass flow cross-sectional area. β is the flow direction with respect to the heading, and it is different from α due to turbulence within the boundary layer. The normal acceleration equals the linear and angular velocities multiplied together, or the normal component of the force divided by the ship's mass (Meriam, 1980) as follows:

$$a_n = \omega_t u = \frac{F_n}{m_{ship}} = \frac{u^2\rho A\sin(\beta)}{m_{ship}}$$
$$\therefore \omega_t = k_2\sin(\beta)u \tag{15}$$

Recall from Figure 11 that $\omega_t = K2(\alpha)u$. Combined with equation 15, it follows that $K2(\alpha) = k_2\sin(\beta)$, where k_2 is a constant term that depends on ρ, A, and m_{ship}. The relationship between α and β is found by fitting the slope values from Figure 11 to equation 15 as shown in Figure 13. The best fit occurs when $\beta = 3\alpha$. Note that 30° of rudder angle (close to the tightest turning circle of the Bay-class ships) corresponds to 90° of water deflection for this model.

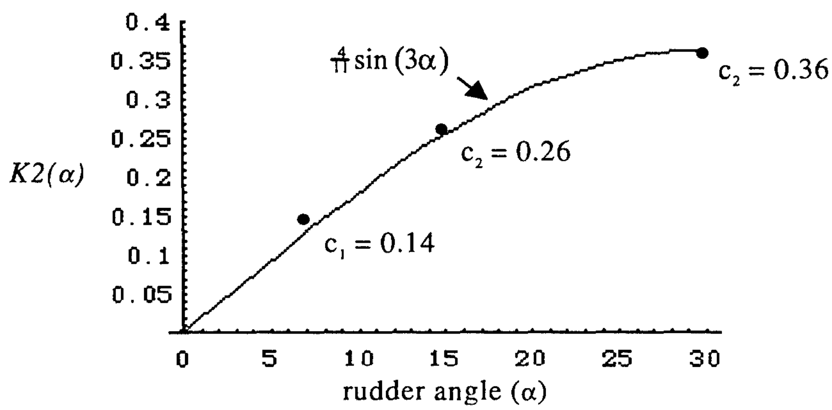

Figure 13 *Simulation Slope Data and Sinusoidal Curve Fit (r = 0.998)*

A similar analysis was done for sea trial data and for port and starboard turning circles. The expressions for $K2(\alpha)$ are given in Table 10. The field trial expression differs only by four percent. The identical gain function was found for port and starboard data.

Table 10 *Expressions for* $K2(\alpha)$

K2(α)	
Simulation	***Sea Trial***
$\frac{4}{11}\sin(3\alpha)$	$\frac{8}{23}\sin(3\alpha)$

Discussion

The functional forms and parameter values were found for the ship's model given in Figure 4. The transfer functions and non-linear gains are summarised in Tables 4, 7, 9, and 10. From Figure 4 and these tables, the time domain equations may be written for the ship's distance and heading for the i^{th} command set (u_i, α_i,t_i) as follows:

$$
\begin{aligned}
d(t-t_i) &= d_o + u_i \frac{a}{b}\left(1+k_1\sin^2\alpha_i\right)^{-0.5}\left[t-t_i-\frac{1}{b}\left(1-e^{-b(t-t_i)}\right)\right] \\
\theta(t-t_i) &= \theta_o + u_i k_2 \sin(3\alpha_i)\left[t-t_i-\frac{1}{c}\left(1-e^{-c(t-t_i)}\right)\right]
\end{aligned}
\tag{16}
$$

Equation 16, along with Table 11, describes a piece-wise linear and continuous model (the non-linear terms are constant for each command). This hybrid model correlates highly with the experimental data ($r > 0.90$). The differences between the MARS VRS and SEA TRIAL parameters are expressed as percentages in Table 11 with respect to the highest parameter value. For example, for the acceleration parameter, a, the difference between MARS VRS and SEA TRIAL is 63% (= (0.068-0.025)/0.068).

Table 11. *Summary of Model Parameters*

	MODEL PARAMETERS			
	MARS VRS		SEA TRIAL	
	acceleration	*deceleration*	*acceleration*	*deceleration*
a	0.025	0.012	0.068 (63%)	0.011 (8.3%)
b	0.024	0.012	0.061 (61%)	0.013 (7.7%)
c	0.025		0.024 (4%)	
k_1	14.5		4.4 (70%)	
k_2	0.36		0.35 (2.8%)	

This section explores the possible uses of the hybrid model outlined in the introduction:

- Compare the MARS VRS software model with the hybrid model,
- Compare the MARS VRS and sea trial data via the hybrid model.
- Note the advantages of the hybrid model for the OOW at sea.

Comparing MARS VRS and Hybrid Models

The ship dynamics equations used in the MARS VRS software code are based on (Gong, 1993), and are used to update the position of virtual ships. These equations were developed from Newton's 2nd law of motion – equilibrium of static and dynamic forces, while the hybrid model considered conservation of momentum and energy for its non-linear terms. Both approaches should yield similar results since Newton's 2nd law can be derived from the fundamental conservation of momentum equation.

The models differ with respect to simplification. That is, the MARS VRS model includes only the primary terms from the full dynamic equations (Gong, 1993), and further simplifies parts of those terms (e.g., $v^{1.8} \rightarrow v^2$). On the other hand, the hybrid model effectively linearises the full dynamics equations, and a linear model exists for every command set

Moreover, if the manoeuvres involve small accelerations so that the linear, first-order terms dominate, then the hybrid model would not be noticeably different from the MARS VRS model, which would not be noticeably different from a ship sailing in calm seas. As the accelerations become more pronounced, the MARS VRS model would deviate from the true behaviour, and the hybrid model parameters would need to be adjusted in order to maintain its high correlation.

Also, the models differ with respect to computational complexity. The MARS VRS requires approximately 300 lines of code to instantiate just the dynamics of the model. The hybrid model would require two lines of code (equation 16) along with declarations, parameter initialisation, input/output function calls, etc., for approximately 30 lines of code.

In summary, the both models produce similar ship behaviours for small acceleration manoeuvres. The clear advantage of the hybrid model is that it would require an order of magnitude fewer lines of code to instantiate into the simulation.

Comparing MARS VRS and SEA TRIAL Data

The hybrid model provides a means for comparing path data generated from the MARS VRS and data generated from the ship at sea. Table 11 lists the two sets of model parameters and their differences expressed as a percentage of the highest value. It is difficult to compare the simulator ship dynamics with the sea ship dynamics because the sea states and payloads are different. With that in mind, it is worth noting the differences, and suggesting possible explanations for these discrepancies.

The time constant τ is the number of seconds to reach 63.2% of the target speed starting from an initial constant speed. During the acceleration phase, τ (= 1/b) is 40 seconds and 14.7 seconds in the simulator and at sea, respectively. The parameter differs by 61%, and the simulated ship takes longer to reach the target speed than the ship at sea. One possible explanation for this large discrepancy may be due to a smaller thrust produced by the simulated ship.

During the deceleration phase, the time constant differs by 7.7%. The time constant depends on the amount of drag on the ship. Recall that the drag force depends on the ship length and the hull's extent into the water. The extent into the water depends on the mass of the ship. Therefore the simulated ship and the ship at sea would seem to have similar mass in order to reproduce a similar un-powered drag force during the deceleration phase.

During the turning circles, the time constant for the target angular velocity ($\tau = 1/c$) differs by 4%, and the behaviour is effectively the same. Even though the real ship sails in rougher seas, the wave motion moves in a single direction more or less. Thus, the accumulated affects of the waves and wake would nearly cancel out during a turning circle.

Recall that the target tangential speed during a turning circle decreases by a factor of $K1(\alpha)$. The parameter k_1 differs by 70% between the MARS VRS and the SEA TRIAL parameter, and the target speed for the simulated ship is significantly lower than the ship at sea. This result is consistent with the explanation that the simulated ship produces less thrust than the ship at sea.

Recall that the target angular velocity during a turning circle decreases by a factor of $K2(\alpha)$. The parameter k_2 differs by 2.8%, which means there is virtually no difference between the simulated ship and the ship at sea.

In summary, the tangential speed behaviour is different between the simulated ship and the ship at sea, but the angular velocity behaviour is similar. It is assumed that simulated ship generates less thrust than the ship at sea, both ships have similar mass, and no significant differences in sea state were inferred by the parameter values.

Hybrid model for Ship at Sea

The hybrid model parameters for the ship at sea reflect the HMCS Thunder ship dynamics on 25 February, 1994 in the Strait of Juan Du Fuca. This sea trial produced a set of turning characteristics for HMCS Thunder, which the crew can reference (Annex C). Previous to this, the turning characteristics tables date from the mid 70's. However, the centre of gravity and moments of inertia would creep over the years from wear and tear, sea barnacles, etc. The daily variations of large payloads (personnel, fuel, supplies, vehicles, etc.) as well as sea states would also significantly impact the ship dynamics on that day.

Knowing the up-to-date dynamic ship behaviour would be advantageous for the OOW. If the ship seems a bit sluggish, then the OOW would want to start the manoeuvre a bit earlier. If the ship were light and slick, then the OOW would want to avoid overshooting the desired heading and speed.

Conceptually, the algorithms in Annex C can be used to generate the turning characteristics tables as path data. Furthermore, the current ship dynamic model can be determined in near real time and simulated onboard. The hybrid model would be initialised with a default set of parameters for a given load and a given sea state. As the ship sails at sea, path data would be continually collected. The computer would parse the data into segments (acceleration profiles and turning circles) and use the techniques described herein to automatically generate an updated set of parameters for the current sea state and loads. Then, the OOW would have access to the most current model of the ship.

The OOW could use the model in a predictive fashion and rehearse manoeuvres through a tight channel, for example, or foresee how the ship behaviour might change after the helicopter is deployed and the missiles are fired. The new model can be added to a library of various sea states and loading conditions, and the comprehensive model can be used for acquisition as well as rehearsal and training activities.

Conclusion

An expression for the ship dynamics was needed to express the OOW's decision-making process as a function of the performance data (Farrell, 2002). The ship dynamics model was identified using simulator and sea trial data. The identification technique yielded a hybrid model (linear differential equations with non-linear parameters) where the simulated ship and the ship at sea are distinguished solely by specific parameter values. The hybrid model is piecewise-linear and continuous. It is mathematically and computationally less complex than the full dynamics equations, but it correlates highly with actual path data.

The hybrid model was compared to the MARS VRS software code. Both models produce similar ship behaviours for small acceleration manoeuvres. However, the hybrid model requires fewer lines of code to instantiate.

The hybrid model was used to compare the MARS VRS and sea trial data. The tangential speed behaviour of the simulated ship is different than the ship at sea, but the angular velocity behaviour is similar amongst the two models. It is assumed that mass and the sea states were similar for both ships, but the simulated ship generated less thrust than the ship at sea.

The hybrid model may be a useful tool for the OOW at sea. A library of parameters can be generated and catalogued that represents different sea states and loading conditions. The OOW can take advantage of the model just as they might use turning characteristic tables. The model can be used effectively for acquisition, rehearsal, and training purposes.

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Annex A

Summary of Manoeuvres and Times

Listed below are the manoeuvres performed at sea.

The first column indicates the manoeuvre. For the first twelve entries, the ship steams straight ahead, (i.e., the rudder position is zero degrees).

The second column contains an estimate of the duration of the manoeuvre based on the on-deck conversation.

The third column contains the manually recorded times. The bracketed quantities represent an alternative time based on the audio tape results (N.B. the conversion factor was $^5/_{18}$ tape counts/sec).

The fourth column converts the tape counter numbers to Greenwich Mean Time. This time is used to locate approximately the manoeuvre in the DGPS data file. These are the start and end times used in the analysis.

The fifth column provides additional comments from the audio tapes that were useful in the locating the manoeuvre in the DGPS data file. For instance, if the bearing was given for a manoeuvre, the location of the manoeuvre could be easily identified on the map of the manoeuvres given in Figure 6.

Note that, there are some fields in the table that appear incomplete or contain a question mark. This indicates that critical pieces of information could not be extracted from the data to complete the field. Also note that during the turning circle data, the second, third and fourth columns have more than two time entries. The middle entries correspond to the time of a completed circle. In some runs, three complete circles before terminating the manoeuvre.

Table A.1 *Summary Of Manoeuvre Times For HMCS Thunder 25 February 1994*

Manoeuvre	***Tape Counter***	***Log times (tape times)***	***GMT (tape)***	***GMT (DGPS)***	***Comments***
	Tape 1				**side A**
0-9 kts	030-060	1:17 (1:48)	17:7:20 17:9:08		steer 090
9-0 0-9 kts	083-174	4:29 (5:28)	17:10:31 17:15:58	17:06:00 17:13:59	Winds NNW 20-25 kts
0-12 kts	331-	1:26?		17:14:00-	525 rpm steer 030
12-0 kts	229-306	4:41 (4:37)	17:19:16 17:23:54	-17:22:59	steer 200 at 383
0-12 kts	418-465	1:26?		17:23:00 17:24:59	repeated with wind on stern
15-0 kts	473-008	(2:13)	17:33:55 17:36:08	17:26:00-	
0-15 kts	008-140	0:45 (7:55)	17:30:29 17:38:24	-17:34:20	**side B**
14-0 kts	165-249	4:49 (5:02)	17:39:54 17:44:56	17:24:24-	2 kts (GPS) at 219
0-14 kts	289-306	1:03 (1:01)	17:48:22 17:47:20	-17:44:34	
10-0 kts	316-379	4:13 (3:47)	17:48:58 17:52:44	17:44:50-	690 rpm
0-10 kts	390-411	1:19 (1:16)	17:53:24 17:54:40		
0-15 kts	460-474	0:45 (0:50)	17:57:36 17:58:26	-18:06:02	430 rpm
	Tape 2				**Side A**
9 kts port 7°	062 -187 (1.5 turns) -220 (2 turns) -250 (2.5 turns) -275 (3 turns)	5:00 (7:30) 4:10 (1:59) (1:48) (1:30)	18:30:59 18:38:29 18:40:28 18:42:16 18:43:46	18:29:00 18:44:00	steer 030
9 kts port 15°	300 -319 -358 -378 -396	2:50 (1:08) (2:20) 2:45 (1:12) (1:05)	18:45:16 18:46:24 18:48:45 18:49:57 18:51:02	18:43:00 18:52:59	steer 200 690 rev (15 knots) at 435
9 kts port 30°	489 -022 -071 -117	2:09 (2:10) (2:56) 2:09 (2:46)	18:56:36 18:58:46 19:03:15 19:06:00	18:57:30 19:05:59	1100 at 0017, **side B**
9 kts stbd 7°	160 -232 -294	4:19 (4:19) 3:45 (3:43)	19:08:35 19:12:54 19:16:37	19:06:50 19:17:59	steer 090 Wind NNW 30 knots

9 kts stbd 15°	345 -386 -420	2:43 (2:28) 2:40 (2:02)	19:19:41 19:22:09 19:24:11	19:18:00 19:24:59	steer 270
9 kts stbd 30°	440 -467 -495	2:12 (1:37) 2:07 (1:41)	19:25:23 19:27:00 19:28:41	19:25:00 19:31:29	steer 340
	Tape 3				**Side A**
12 kts port 7°	050 -117 -172	3:02 (4:01) 2:65 (3:18)	19:23:32 19:27:33 19:30:51	19:32:31 19:40:59	Winds NNW 25 kts steer 160
12 kts port 15°	209 -245 -278	2:03 (2:10) 2:01 (1:59)	19:33:04 19:35:14 19:37:13	19:41:00 19:46:59	steer 010
12 kts port 30°	299 -322	1:34 (1:23) 1:17	19:38:28 19:39:51	19:46:30 19:52:29	Winds NNW 25-30 kts steer 090
12 kts stbd 7°	382 -420 -455	2:45 (2:17) 2:45 (2:06)	19:43:27 19:45:44 19:47:50	19:52:00 19:59:59	steer 030
12 kts stbd 15°	480 -503	2:05 (1:23) 1:55	19:49:20 19:50:43	20:00:00 20:06:30	steer 120
12 kts stbd 30°	(N/A)	1:32 ?		20:07:00 20:13:59	
	Tape 4				**Side A**
15 kts port 7°	035 -094 -144	2:34 (3:32) 2:20 (3:00)	20:37:25 20:40:57 20:43:57	20:37:00 20:44:59	steer 190
15 kts port 15°	177 -213 -240	1:10 (2:10) 1:41 (1:37)	20:45:56 20:48:06 20:49:43	20:45:00 20:50:59	steer 90
15 kts port 30°	271 -293 -312	1:26 (1:19) 1:15 (1:08)	20:51:35 20:52:54 20:54:02	20:50:00 20:54:59	steer 350
15 kts stbd 7°	333 -363 -395	(1:48) (1:55)	20:55:18 20:57:06 20:59:01	20:54:30-	steer 270 midships was called unexpectedly run aborted at 395
15 kts stbd 7°	418 -449 -477	2:29 (1:52) 2:15 (1:41)	21:00:24 21:02:15 21:03:56	-21:07:59	steer 010
15 kts stbd 15°	495	1:45 1:36	21:05:01	21:07:30 21:11:59	steer 090 tape ran out. no comments after this run
15 kts stbd 30°	(N/A)	1:10 1:05		21:12:00 21:17:59	

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Annex B

Derived Data Equations

All position, heading, and velocity data for the manoeuvres were derived from latitude (lat) and longitude (lng) data. Converting the raw data files – degrees and minutes – to degrees only is done as follows:

$$\begin{aligned} \text{lat} &= 48 + \frac{\text{column 4} - 4800}{60} \\ \text{lng} &= -\left(123 + \frac{\text{column 5} - 12300}{60}\right) \end{aligned} \tag{B.1}$$

The derived data take into consideration the distance between two points (1 and 2) along a spherical earth (although a flat earth model is just as valid for the distances travelled during the manoeuvres). The distance between two points (p_1 and p_2) on a spherical earth with radius, R, is the length of an arc that subtends those points, or:

$$p_2 - p_1 = R\theta \tag{B.2}$$

The angle, θ, is related to the straight line distance between the two points via an inverse sine relationship as follows:

$$\theta = 2\arcsin\left(\frac{d}{2R}\right) \tag{B.3}$$

The distance, d, is also the hypotenuse of a right angle triangle whose 90° vertex is the intersection of line of latitude for point 1 and line of longitude for point 2 (or visa versa), or:

$$d^2 = x^2 + y^2 \tag{B.4}$$

The distance, x, is related to the longitudinal difference between the two points and the circular radius, r, for that line of latitude as follows:

$$\begin{aligned} x &= 2r\sin\left(\frac{\text{lng}_2 - \text{lng}_1}{2}\right) \\ &= 2R\cos(\text{lat}_1)\sin\left(\frac{\text{lng}_2 - \text{lng}_1}{2}\right) \end{aligned} \tag{B.5}$$

The distance, y, is related to the latitude difference between the two points as follows:

$$y = 2R\sin\left(\frac{lat_2 - lat_1}{2}\right) \tag{B.6}$$

Substituting the equations into equation B.2 and simplifying yields the expression for the magnitude between two points in terms of the longitude and latitude differences as follows:

$$p_2 - p_1 = 2R\sin^{-1}\left(\sqrt{\cos^2(lat_1)\sin^2\left(\frac{lng_2 - lng_1}{2}\right) + \sin^2\left(\frac{lat_2 - lat_1}{2}\right)}\right) \tag{B.7}$$

Points 1 and 2 represent the **linear distance** between two subsequent points on a single trajectory. The same equation is used to calculated the **linear position error** between two separate ship trajectories (a and b) at a single point in time as follows:

$$p_b - p_a = 2R\sin^{-1}\left(\sqrt{\cos^2(lat_a)\sin^2\left(\frac{lng_b - lng_a}{2}\right) + \sin^2\left(\frac{lat_b - lat_a}{2}\right)}\right) \tag{B.8}$$

The ships' **tangential velocities** are calculated by dividing equation B.7 by the time interval, dt, as follows:

$$v_a = \frac{2R}{dt}\sin^{-1}\left(\sqrt{\cos^2(lat_{a1})\sin^2\left(\frac{lng_{a2} - lng_{a1}}{2}\right) + \sin^2\left(\frac{lat_{a2} - lat_{a1}}{2}\right)}\right) \tag{B.9}$$

$$v_b = \frac{2R}{dt}\sin^{-1}\left(\sqrt{\cos^2(lat_{b1})\sin^2\left(\frac{lng_{b2} - lng_{b1}}{2}\right) + \sin^2\left(\frac{lat_{b2} - lat_{b1}}{2}\right)}\right) \tag{B.10}$$

For the simulator trials, dt = 5 seconds and dt = 1 second for the sea trials. Now, the **relative velocity** between the two trajectories is calculated by substituting equations B.9 and B.10 into the following equation:

$$\|v_b - v_a\| = \sqrt{v_a^2 + v_b^2 - 2v_a v_b \cos(\theta_b - \theta_a)} \tag{B.11}$$

The **relative heading**, θ_b-θ_a, is required to fully evaluate equation B.11. Two consecutive points are required to calculate the ship's heading. From the quantities in equations B.4, B.5, and B.6, the hypotenuse direction is related to the inverse tangent of the ratio of y and x, or:

$$\theta = \tan^{-1}\left(\frac{\sin\left(\frac{lat_2 - lat_1}{2}\right)}{\cos(lat_1)\sin\left(\frac{lng_2 - lng_1}{2}\right)}\right) \tag{B.12}$$

Therefore, the relative heading is a simple subtraction, and the expression is as follows:

$$\theta_b - \theta_a = \tan^{-1}\left(\frac{\sin\left(\frac{lat_{b2} - lat_{b1}}{2}\right)}{\cos(lat_{b2})\sin\left(\frac{lng_{a2} - lng_{a1}}{2}\right)}\right) - \tan^{-1}\left(\frac{\sin\left(\frac{lat_{a2} - lat_{a1}}{2}\right)}{\cos(lat_{a1})\sin\left(\frac{lng_{a2} - lng_{a1}}{2}\right)}\right) \quad \text{(B.13)}$$

Similarly, a difference formula can be used to generate the **relative angular velocity**. Since the data is discrete, the relative angular velocity is calculated by backward, forward, or central difference equations as follows:

$$\omega_b - \omega_a = (\theta_b - \theta_a)_n - (\theta_b - \theta_a)_{n-1}$$

or

$$\omega_b - \omega_a = (\theta_b - \theta_a)_{n+1} - (\theta_b - \theta_a)_n \quad \text{(B.13)}$$

or

$$\omega_b - \omega_a = \frac{(\theta_b - \theta_a)_{n+1} - (\theta_b - \theta_a)_{n-1}}{2}$$

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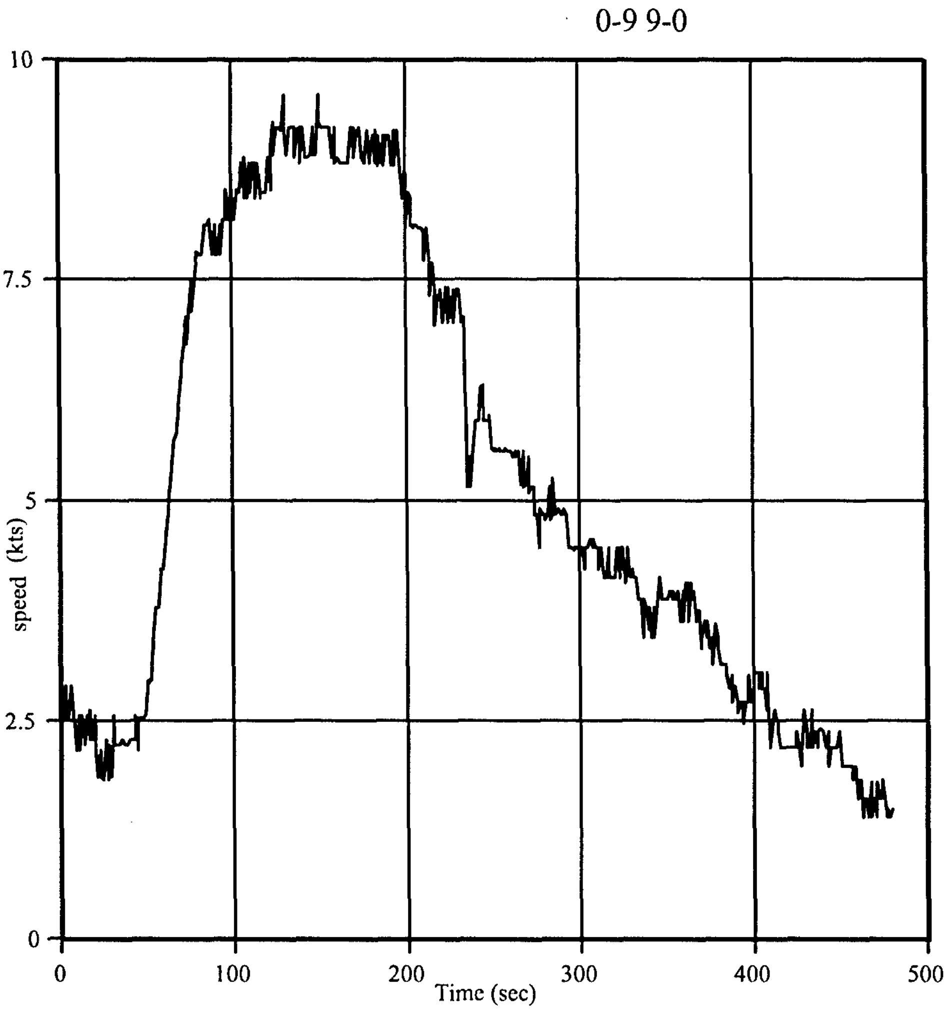
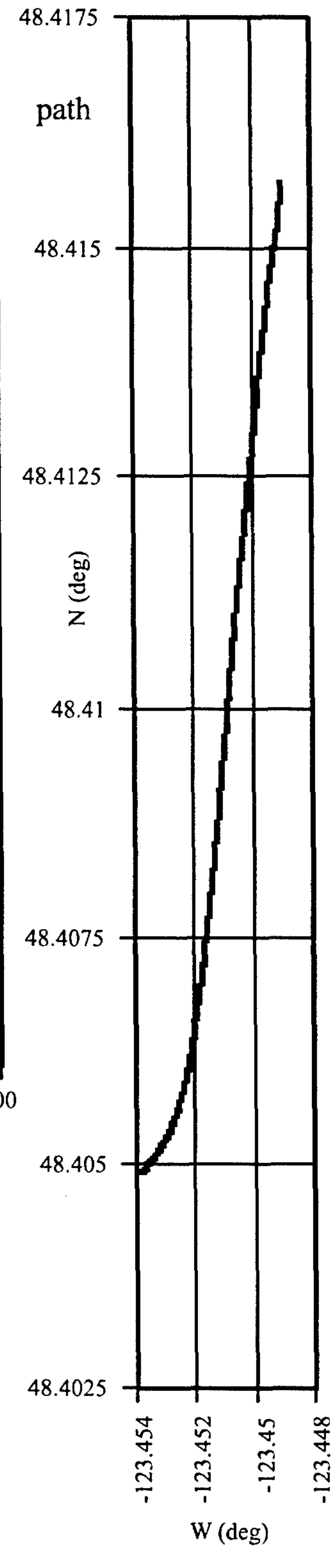
Annex C

Acceleration Profiles

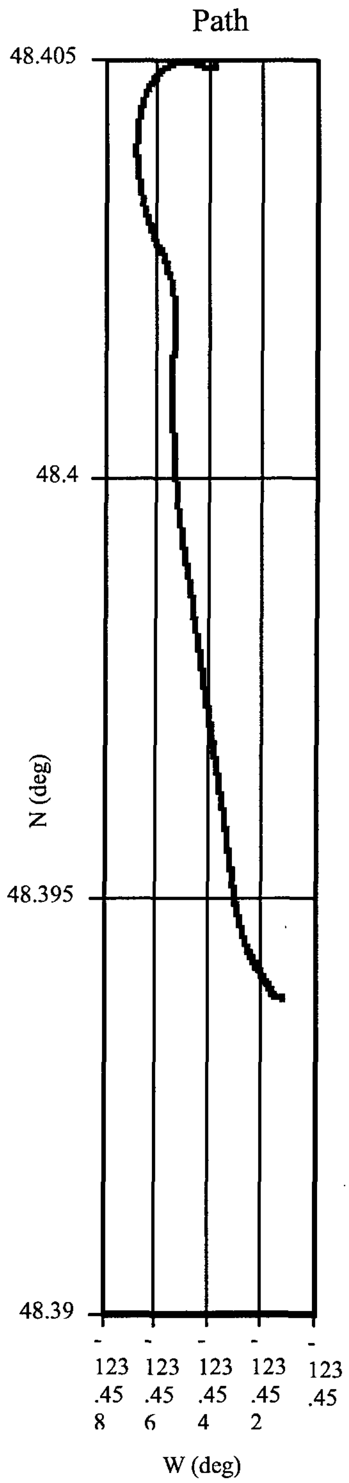
The following graphs show the path of the ship while performing the acceleration profiles on 25 February 1994 in the Strait of Juan Du Fuca. The derived data are the linear speed, and this is also plotted for each manoeuvre. Note that the speed plots are noisey. The points generated every second for the sea trials and every five seconds for the simulator trials.

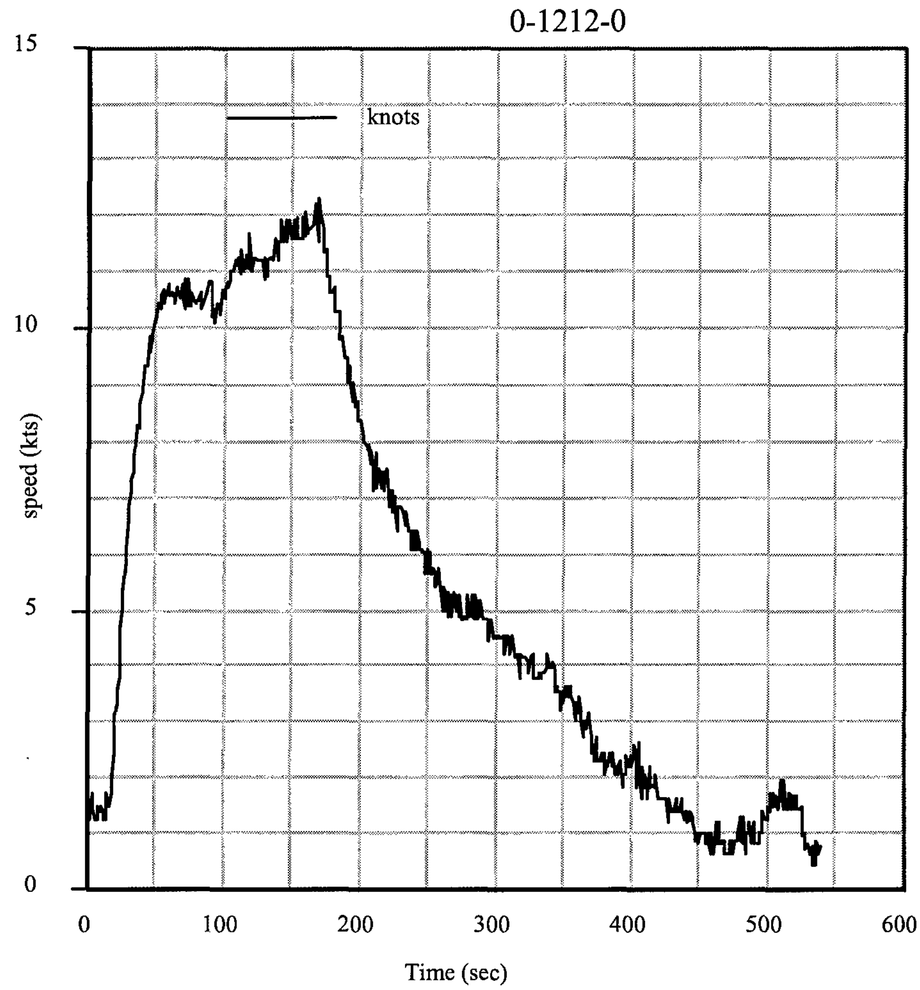


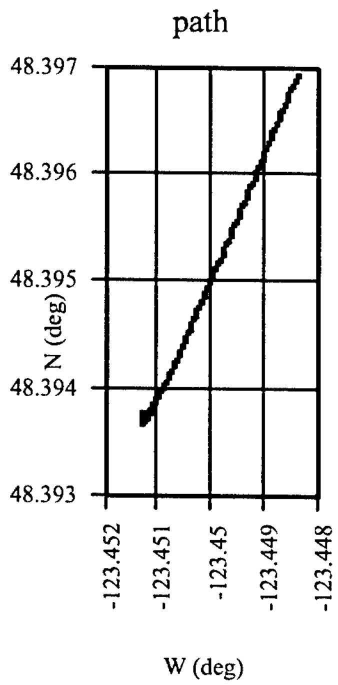
0-9 9-0
speed (kts)
10
7.5
5
2.5
0
0
100
200
300
400
500
Time (sec)



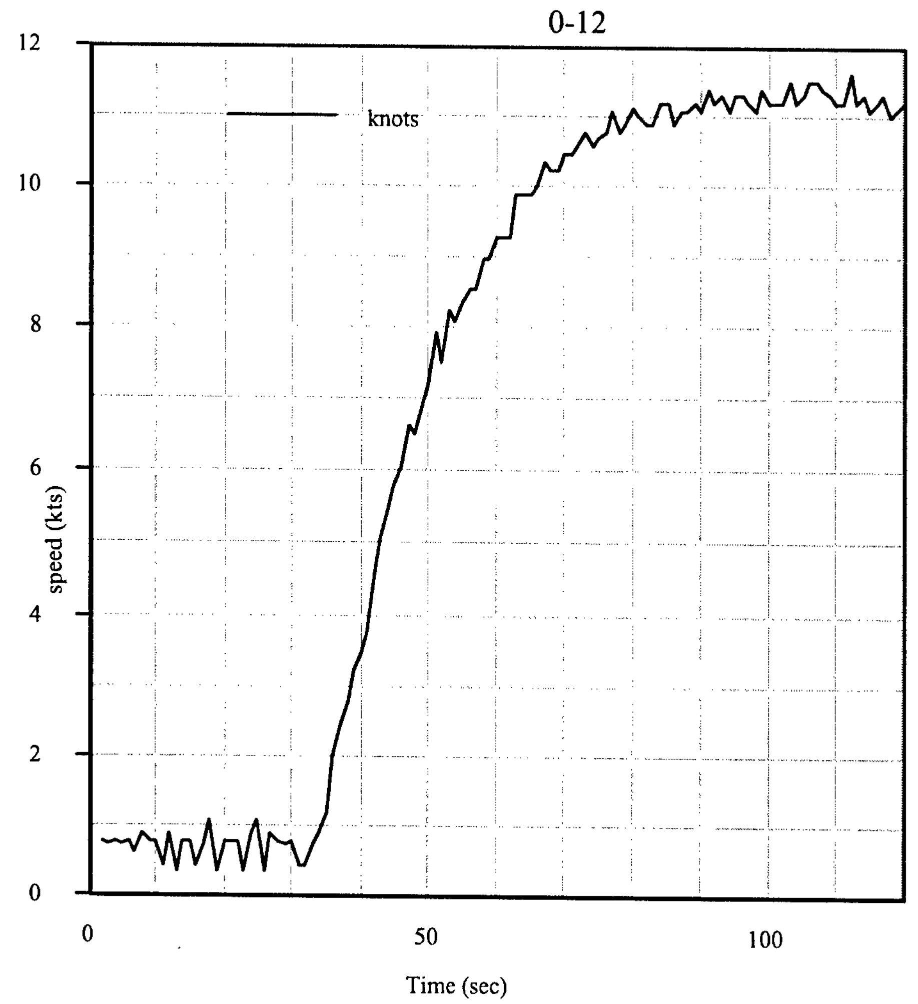
path
48.4175
48.415
48.4125
48.41
48.4075
48.405
48.4025
N (deg)
-123.454
-123.452
-123.45
-123.448
W (deg)


Path
48.405
48.4
48.395
48.39
N (deg)
-123.458
-123.456
-123.454
-123.452
-123.45
W (deg)

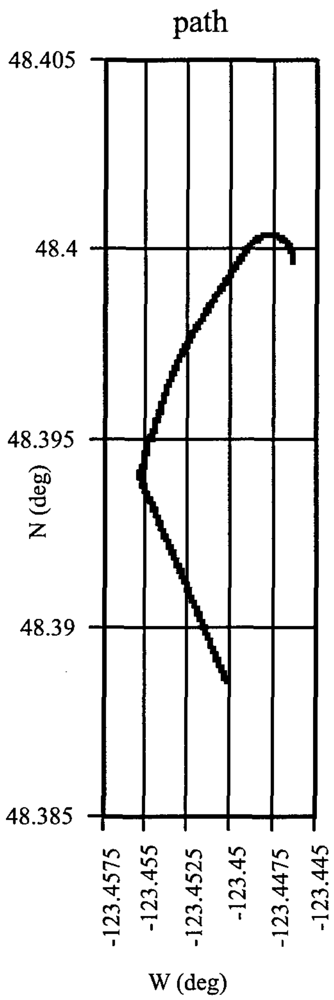

0-1212-0
knots
speed (kts)
15
10
5
0
0
100
200
300
400
500
600
Time (sec)

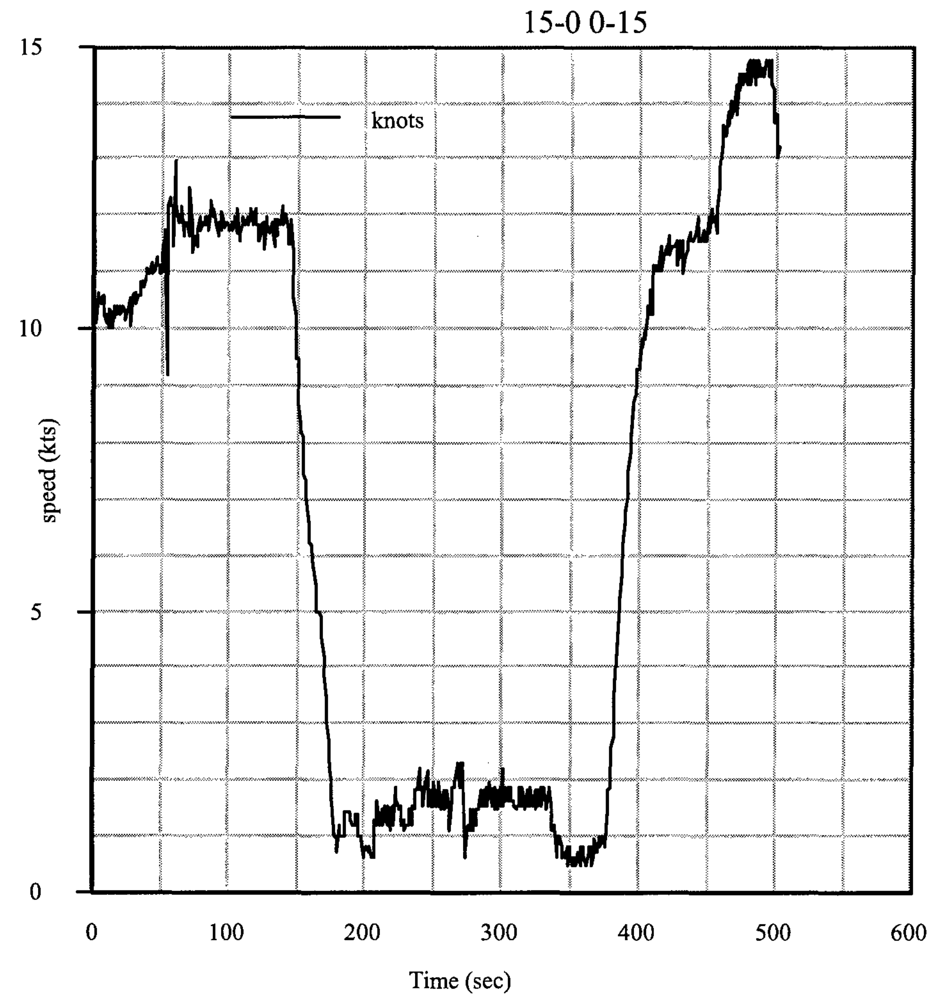


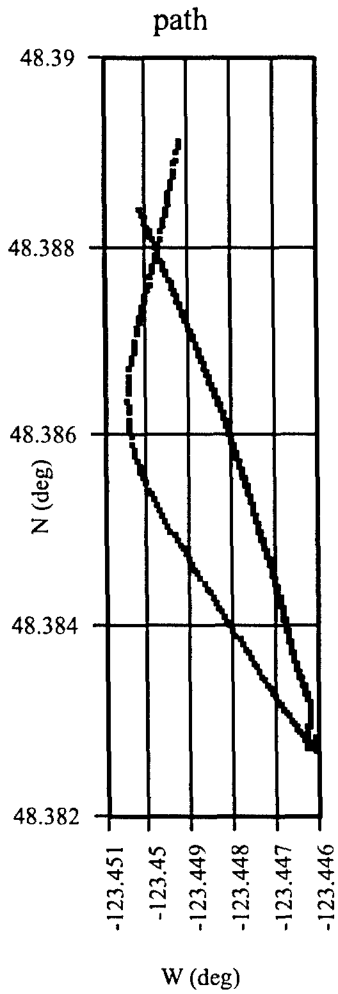
path
48.397
48.396
48.395
48.394
48.393
N (deg)
-123.452
-123.451
-123.45
-123.449
-123.448
W (deg)

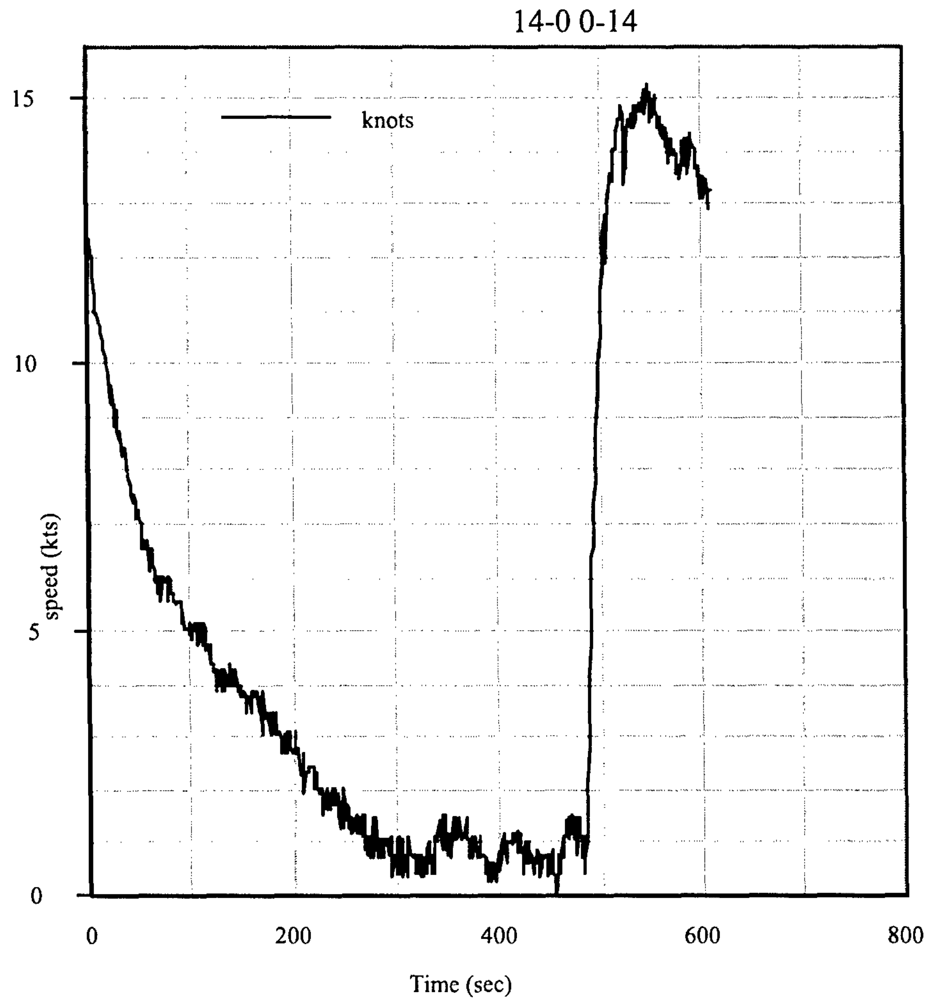


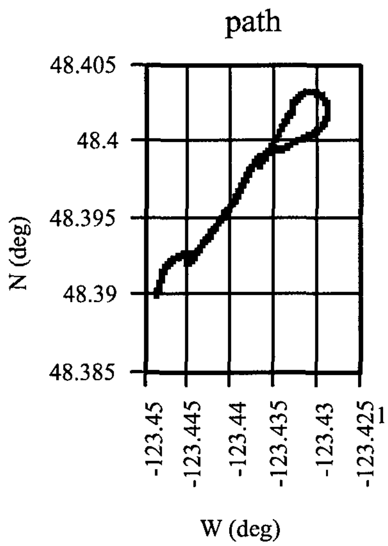
0-12
knots
12
10
8
6
4
2
0
speed (kts)
0
50
100
Time (sec)


path
48.405
48.4
48.395
48.39
48.385
N (deg)
-123.4575
-123.455
-123.4525
-123.45
-123.4475
-123.445
W (deg)

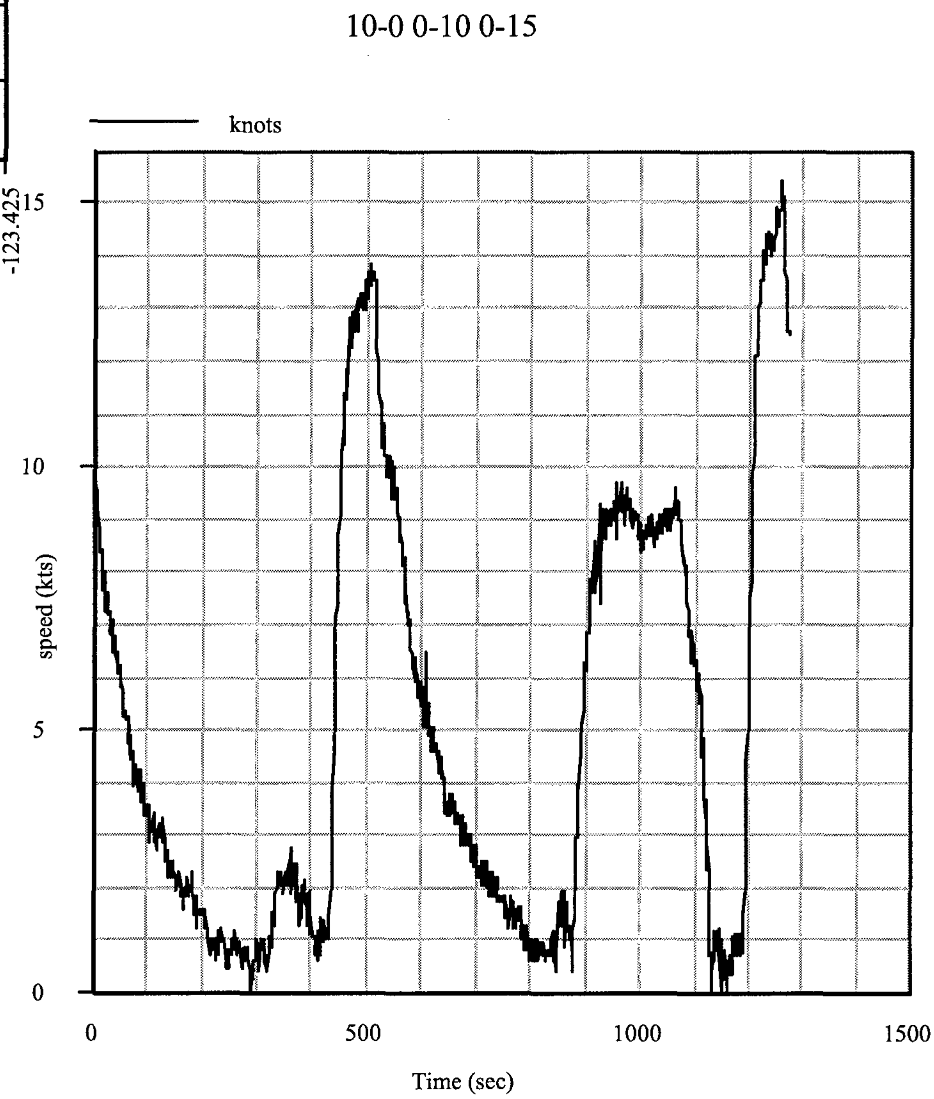

15-0 0-15
knots
15
10
5
0
speed (kts)
0
100
200
300
400
500
600
Time (sec)


path
48.39
48.388
48.386
48.384
48.382
N (deg)
-123.451
-123.45
-123.449
-123.448
-123.447
-123.446
W (deg)


14-0 0-14
knots
15
10
5
0
speed (kts)
0
200
400
600
800
Time (sec)



path
48.405
48.4
48.395
48.39
48.385
N (deg)
-123.45
-123.445
-123.44
-123.435
-123.43
-123.425
W (deg)



10-0 0-10 0-15
knots
15
10
5
0
speed (kts)
0
500
1000
1500
Time (sec)

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Annex D

Turning Circles

Listed in this Annex are the turning circle data for the HMCS Thunder on 25 February 1994 in the Strait of Juan Du Fuca. The amount of turn is expressed as a bearing (degrees). The advance yards and the transfer yards are the distances the ship travels from a starting position in a forward and perpendicular direction, respectively. Also, the table lists the vessel speed during the turn. Note that the speed decreases as the ship turns. The average turn rate (average angular velocity) is given at the end of each table. Also included at the end of the table is the time to turn and an insert graph of the path of the ship.

The Table format is identical to a set of tables for the HMCS Fundy generated in 1978. The particular document is confidential. However, a computer algorithm, that used the raw data and formatted the tables, is listed below. Note that the average turn rate is calculated directly from the time to turn data and not from the computer programme in this case.

turning.c

```
// CALCULATING GMT from TAPE TIMES
// NOTE THAT THE STBD FILES HAVE BEEN FLIPPED 180 deg TO RESEMBLE PORT
FILES
// THIS PROGRAM IS GOOD ONLY FOR POSITIVELY INCREASING CARTESIAN
HEADING ANGLES

#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <StandardFile.h>
#include <Files.h>
#define PI 3.141592653589793
#define EarthRadius 6378388*1.093613298 // converted from metres to yards
FILE *infile, *outfile, *outfile1;

void main()
{
double advance;          // yards
double transfer; // yards
double time;             // hms
double west;             // degrees
double north;            // degrees
double speed;            // knots
double heading = 0;      // degrees inputted as 0 degrees true north, converted to cartesian
angles.
double oldheading;       // degrees
static double initwest, initnorth, inithead, inittime;        // initial values
static double avgomega = 0;
static int j = 0;
static int k = 0;
static int m = 0;
static int count = 0;
static int delta = 0;    // increment in angular displacement
static int flag = 0;
double dist;             // distance from starting point (yards)
double x;                // latitudinal distance from starting point
double y;                // longitudinal distance from starting point
double phi;              // angular displacement from initial heading
double storetime[132];
double sec();
void printtime();
int i, n;

infile = fopen("in","r");
outfile = fopen("out","w");
outfile1 = fopen("out1","w");
```

```
// store starting point
if(!(fscanf(infile,"%lf %lf %lf %lf %lf%c", &time, &initwest, &initnorth, &speed,
&heading)))
{fprintf(outfile1,"What happened?"); abort();}     // get new values
inittime = sec(time);

for (i = 1;!feof(infile);i++)
{
        oldheading = heading;
        fscanf(infile,"%lf %lf %lf %lf %lf%c", &time, &west, &north, &speed, &heading);
        // get new values

        heading = 90 - heading;                                          // use
cartesian coordinates
        if (heading < 0) heading += 360;                                 // use postive
cartesian angles
        if (flag == 0) {flag = 1; inithead = heading;}                   // set initial heading

        if (heading - oldheading > 0.001 && heading - oldheading < 180)          // turning
rate calculation
        {avgomega += heading - oldheading; count++;}

//      fprintf(outfile1,"%lf\t%lf\t%lf\t%lf\t%d\n", time, west, north, heading, delta);
//      if (j == 1) {for (n=1;n <= m;n++) printtime( sec(storetime[n]) - inittime ,4); break;}

        if (heading < inithead + delta - 15)
        if (inithead + delta - 15 - heading > 15) heading += 360; else ;// noise


        if (heading >= inithead + delta)
        {
                x = cos(initnorth*PI/180)*sin((west-initwest)/2*PI/180);
                y = sin((north-initnorth)/2*PI/180);
                dist = 2*EarthRadius*sqrt(x*x + y*y);
                phi = atan2(y,x) - inithead*PI/180;
                advance = dist*cos(phi);
                transfer = dist*sin(phi);

                if (k == 6)       {m++; storetime[m] = time; k = 0;} k++;          //turning
times every 90 degrees

                if (delta >= 15)
                fprintf(outfile,"%d\t%6.1lf\t%6.1lf\t%5.1lf\n",delta,advance,transfer,speed);

                if (delta == 360)
                {
```

```
                        fprintf(outfile,"\naverage turn rate = %.2lf Deg/Sec
\n\n",avgomega/count);
                        fprintf(outfile,"Time to turn\n");
                        for (n=1;n <= m;n++) printtime( sec(storetime[n]) - inittime ,4);
                        m = 0;
                        inittime = sec(time);

                        fprintf(outfile,"\f");
                        delta = 0; j += 1;              // identify 360 degrees of turning
                }

                if (delta == 0)
                {
                        fprintf(outfile,"HMCS THUNDER TURNING
CHARACTERISTICS\n\n");
                        fprintf(outfile,"AT 9 KNOTS STARBOARD 15 DEGREES OF
HELM (circle %d)\n\n\n", j+1);
                        fprintf(outfile,"AMOUNT OF TURN DEGREES\tADVANCE
YARDS\tTRANSFER YARDS\tSPEED KNOTS\n\n");

        fprintf(outfile,"%d\t%6.1lf\t%6.1lf\t%5.1lf\n",delta,advance,transfer,speed);
                }

                delta += 15;
        }

}

fclose(infile);
fclose(outfile);
fclose(outfile1);

}
```

sec.c

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <StandardFile.h>
#include <Files.h>
double sec(hms)
double hms;
{
double seconds;
// printf("%lf\n",hms);
seconds = floor(hms/10000)*60*60 +
floor((hms - floor(hms/10000)*10000)/100)*60 +
(hms - floor(hms/10000)*10000) - floor((hms - floor(hms/10000)*10000)/100)*100;
```

```
// fprintf(outfile,"%.0lf:%.0lf\n",min,sec);
return seconds;
}
```

printtime.c

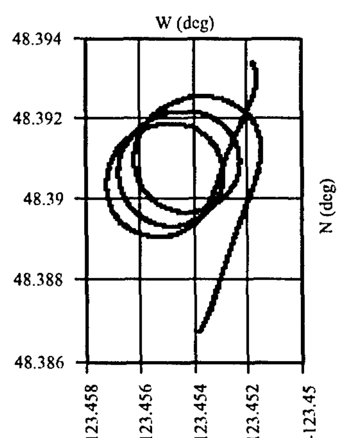
```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <StandardFile.h>
#include <Files.h>
extern FILE *outfile, *outfile1;

void printtime(totalsec,display)
double totalsec;
int display;
{
double hours;
double minutes;
double seconds;
static int i = 0;

hours = totalsec/60.0/60.0;
minutes = (hours - floor(hours))*60.0;
seconds = (minutes - floor(minutes))*60.0;
minutes = floor(minutes);
hours = floor(hours);
if (display == 1)
printf("diff = %.0lf:%.0lf:%.0lf\n",hours,minutes,seconds);
if (display == 2)
printf("diff = %.0lf hrs %.0lf mins %.0lf sec \n",hours,minutes,seconds);
if (display == 3)
printf("diff = %.0lf min %.0lf sec \n",minutes,seconds);

if (display == 4)
{
        if (i == 4) i = 0; i++;
        fprintf(outfile,"%d\t%.0lf min %.0lf sec \n",90*i,minutes,seconds);
        fprintf(outfile1,"%d\t%.0lf min %.0lf sec \n",90*i,minutes,seconds);
}
}
```

HMCS THUNDER TURNING CHARACTERISTICS

AT 9 KNOTS PORT 7 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	4.7	0.0	8.4
15	139.0	10.2	8.3
30	177.0	25.4	8.1
45	220.7	57.9	8.1
60	241.7	88.3	8.2
75	254.4	122.2	8.0
90	259.6	177.1	8.3
105	249.7	227.6	8.4
120	230.4	270.8	8.3
135	195.0	314.3	8.3
150	149.7	348.2	8.5
165	110.1	365.1	8.8
180	57.6	372.5	8.4
195	6.5	365.2	8.3
210	-36.3	345.7	8.8
225	-65.5	320.8	8.8
240	-87.6	289.6	8.5
255	-101.6	256.0	7.8
270	-106.9	206.5	8.1
285	-98.8	162.4	7.9
300	-84.0	131.2	7.6
315	-59.5	101.0	7.8
330	-24.6	74.5	7.7
345	16.1	59.3	7.7
360	59.4	55.3	7.6

average turn rate = 1.28 Deg/Sec



Time to turn
90 1 min 19 sec
180 2 min 24 sec
270 3 min 20 sec
360 4 min 17 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 9 KNOTS PORT 7 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	59.4	55.3	7.6
15	93.3	61.3	7.6
30	128.2	76.8	7.4
45	155.3	99.2	7.6
60	178.9	132.2	8.0
75	191.7	166.0	8.0
90	198.0	215.9	8.1
105	191.2	256.3	8.2
120	171.6	302.6	8.0
135	138.0	345.9	8.0
150	96.6	376.9	8.3
165	48.1	396.3	8.2
180	-3.6	403.1	8.4
195	-45.2	397.1	8.3
210	-87.4	378.2	8.2
225	-116.3	355.0	8.4
240	-141.4	321.8	8.1
255	-157.8	279.6	8.2
270	-161.9	235.0	8.1
285	-153.6	191.1	7.7
300	-140.6	164.0	7.8
315	-109.8	127.3	8.0
330	-85.1	109.2	7.8
345	-45.0	92.6	8.1
360	-2.6	87.7	7.9

average turn rate = 1.59 Deg/Sec

Time to turn

90	0 min 53 sec
180	1 min 58 sec
270	2 min 54 sec
360	3 min 49 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 7 KNOTS PORT 9 DEGREES OF HELM (circle 3)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	-2.6	87.7	7.9
15	39.8	95.1	7.6
30	76.2	111.9	8.1
45	107.3	136.4	7.6
60	130.3	168.0	7.4
75	143.2	201.6	8.4
90	148.7	242.6	8.1
105	140.0	290.9	8.2
120	122.8	327.7	8.3
135	89.3	372.0	8.3
150	42.5	405.3	8.6
165	-2.3	421.5	8.5
180	-44.4	425.8	8.4
195	-95.6	417.8	8.3
210	-129.7	402.7	8.1
225	-169.1	371.4	8.1
240	-187.8	345.6	8.1
255	-201.0	312.5	7.8
270	-205.8	267.3	8.1
285	-197.9	222.2	8.2
300	-184.0	189.9	7.8
315	-153.8	151.1	8.0
330	-114.3	123.2	7.9
345	-86.3	113.1	7.7
360	11.9	104.8	8.4

average turn rate = 1.57 Deg/Sec

Time to turn

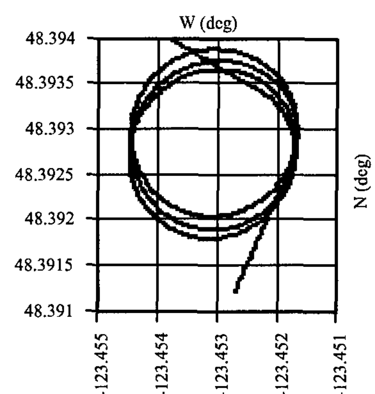
90	0 min 54 sec
180	1 min 57 sec
270	2 min 52 sec
360	4 min 1 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 9 KNOTS PORT 15 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	5.1	0.0	9.0
15	202.6	2.0	8.4
30	232.0	13.2	8.0
45	260.1	34.1	7.5
60	277.9	58.4	7.5
75	287.7	86.1	7.6
90	289.4	112.0	7.6
105	286.6	133.2	7.5
120	274.1	165.7	7.8
135	259.1	186.5	7.5
150	232.3	207.6	7.4
165	200.8	220.6	7.8
180	175.7	222.9	7.4
195	142.2	217.4	7.6
210	118.5	206.8	7.6
225	97.8	190.8	7.8
240	74.6	159.6	7.5
255	67.0	139.9	7.6
270	63.8	110.5	7.5
285	70.3	74.2	7.5
300	79.0	56.1	7.3
315	97.1	33.7	7.3
330	126.6	12.1	7.2
345	145.5	5.1	7.2
360	185.4	0.8	7.3

average turn rate = 1.54 Deg/Sec



Time to turn
90 1 min 17 sec
180 1 min 58 sec
270 2 min 39 sec
360 3 min 24 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 9 KNOTS PORT 15 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	185.4	0.8	7.3
15	209.5	4.8	7.3
30	235.5	16.3	7.3
45	253.7	31.9	7.4
60	267.3	51.2	6.8
75	275.3	74.1	7.4
90	277.6	98.9	7.5
105	271.9	132.3	7.7
120	260.8	156.1	8.0
135	241.9	179.8	7.9
150	213.4	200.5	8.0
165	189.4	210.5	7.8
180	155.4	215.1	7.4
195	125.3	210.0	8.0
210	101.8	199.4	7.6
225	78.4	180.4	7.9
240	61.1	156.0	7.8
255	51.6	132.0	7.6
270	48.1	106.9	7.3
285	53.3	70.4	7.2
300	63.0	48.6	7.0
315	85.5	21.2	6.9
330	105.0	7.7	7.0
345	131.5	-2.4	7.6
360	164.9	-5.5	7.4

average turn rate = 2.29 Deg/Sec

Time to turn

90	0 min 37 sec
180	1 min 20 sec
270	1 min 59 sec
360	2 min 44 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 9 KNOTS PORT 15 DEGREES OF HELM (circle 3)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	164.9	-5.5	7.4
15	197.5	-0.2	7.2
30	223.3	12.7	7.4
45	235.3	23.4	7.1
60	252.9	46.1	7.3
75	263.6	72.9	7.2
90	266.5	97.5	7.5
105	261.7	126.4	7.5
120	244.6	161.3	7.8
135	230.4	177.6	7.7
150	205.7	195.8	7.8
165	168.2	210.0	7.9
180	146.2	211.9	7.8
195	111.4	206.8	7.6
210	88.0	196.1	7.6
225	64.8	177.2	7.8
240	48.0	153.2	7.3
255	38.0	125.7	7.4
270	35.2	96.9	7.3
285	41.3	65.0	7.2
300	50.9	46.4	7.8
315	67.0	26.6	7.7
330	94.4	6.1	7.8
345	125.8	-6.0	7.2
360	158.7	-9.1	7.4

average turn rate = 2.00 Deg/Sec

Time to turn

90	0 min 39 sec
180	1 min 21 sec
270	2 min 3 sec
360	2 min 46 sec

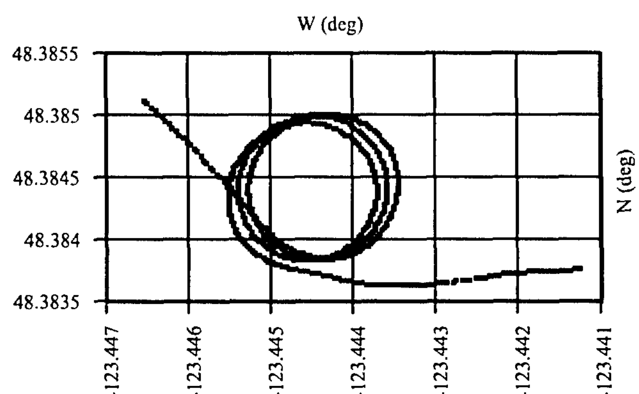
HMCS THUNDER TURNING CHARACTERISTICS

AT 9 KNOTS PORT 30 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	4.8	0.0	8.5
15	186.9	-6.0	7.5
30	204.6	2.9	6.8
45	215.8	12.4	6.5
60	226.1	26.6	6.0
75	232.9	45.7	6.2
90	234.0	62.2	5.8
105	231.1	78.6	5.7
120	221.3	100.2	6.2
135	210.6	113.3	5.8
150	192.1	126.6	5.7
165	179.6	130.7	6.1
180	162.4	131.9	6.5
195	144.9	128.7	6.3
210	125.8	119.4	6.6
225	114.4	109.9	6.7
240	101.4	92.8	6.1
255	93.9	72.9	6.0
270	91.5	51.6	6.4
285	95.6	30.5	6.4
300	105.4	11.6	6.2
315	112.1	3.7	5.8
330	128.4	-8.3	5.8
345	147.5	-14.4	5.9
360	167.0	-15.7	5.6

average turn rate = 1.87 Deg/Sec

Time to turn
90 1 min 5 sec
180 1 min 38 sec
270 2 min 11 sec
360 2 min 44 sec



HMCS THUNDER TURNING CHARACTERISTICS

AT 9 KNOTS PORT 30 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	167.0	-15.7	5.6
15	179.1	-13.6	5.4
30	196.2	-6.6	5.4
45	210.8	5.0	5.3
60	220.1	17.8	5.6
75	226.6	36.0	5.8
90	228.4	55.3	5.8
105	225.2	71.2	5.7
120	215.2	92.3	6.2
135	206.6	102.6	5.8
150	190.5	114.8	6.2
165	171.0	121.6	5.9
180	153.5	122.6	6.2
195	136.4	119.5	6.2
210	117.6	110.0	6.3
225	106.6	100.5	6.5
240	94.1	82.5	6.4
255	87.4	65.5	6.4
270	85.2	44.1	6.3
285	88.8	23.1	6.4
300	98.2	4.7	6.2
315	107.4	-5.3	5.8
330	121.0	-15.2	5.7
345	139.6	-22.2	6.1
360	156.2	-23.4	6.0

average turn rate = 2.74 Deg/Sec

Time to turn

90	0 min 33 sec
180	1 min 6 sec
270	1 min 38 sec
360	2 min 10 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 9 KNOTS PORT 30 DEGREES OF HELM (circle 3)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	156.2	-23.4	6.0
15	175.1	-20.0	5.8
30	192.1	-11.8	5.6
45	203.6	-1.6	5.3
60	214.2	13.7	5.8
75	220.3	31.8	5.8
90	221.8	47.5	5.5
105	219.3	62.7	5.3
120	212.1	79.8	5.5
135	202.2	92.2	5.8
150	184.1	105.6	5.7
165	168.3	110.8	6.1
180	154.3	112.0	6.0
195	137.5	109.4	6.2
210	118.6	101.0	6.3
225	102.4	87.4	6.5
240	90.3	68.7	6.7
255	84.7	51.4	6.6
270	83.1	29.4	6.3
285	86.4	11.7	6.3
300	94.1	-4.1	6.2
315	104.9	-17.3	6.1
330	121.4	-28.6	5.8
345	139.8	-34.7	5.8
360	152.7	-35.3	5.6

average turn rate = 2.71 Deg/Sec

Time to turn

90	0 min 34 sec
180	1 min 6 sec
270	1 min 40 sec
360	2 min 11 sec

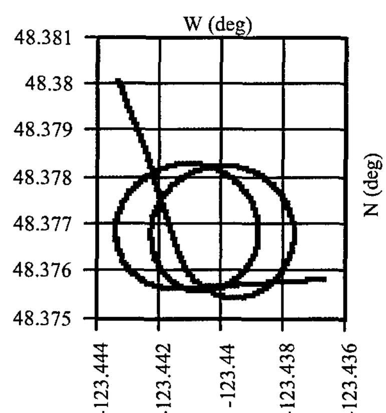
HMCS THUNDER TURNING CHARACTERISTICS

AT 12 KNOTS PORT 7 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	6.5	0.0	11.5
15	336.5	0.8	11.1
30	375.1	18.0	10.6
45	412.7	48.2	10.7
60	433.1	77.1	10.2
75	446.4	115.2	10.4
90	448.6	144.1	10.5
105	442.0	184.3	10.4
120	425.0	222.6	11.1
135	398.3	255.5	11.1
150	363.9	281.1	11.0
165	311.3	300.4	11.3
180	267.0	302.9	11.2
195	229.2	295.7	11.5
210	183.0	274.6	11.4
225	143.5	241.1	11.6
240	118.5	203.4	11.6
255	103.3	160.3	11.8
270	97.6	101.1	11.8
285	107.5	50.0	11.4
300	130.6	4.1	11.2
315	154.4	-24.7	10.7
330	189.8	-49.5	10.9
345	224.5	-62.5	10.9
360	278.8	-67.3	10.6

average turn rate = 1.40 Deg/Sec

Time to turn
90 1 min 26 sec
180 2 min 10 sec
270 2 min 55 sec
360 3 min 39 sec



HMCS THUNDER TURNING CHARACTERISTICS

AT 12 KNOTS PORT 7 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	278.8	-67.3	10.6
15	313.5	-60.5	10.4
30	350.2	-42.6	10.4
45	371.2	-24.0	10.0
60	387.7	-1.2	10.2
75	401.4	36.4	10.6
90	404.2	70.9	10.2
105	396.8	110.7	10.4
120	381.7	142.4	10.7
135	355.7	174.5	10.7
150	321.5	199.1	10.6
165	280.8	214.1	11.2
180	236.8	218.1	11.1
195	192.7	210.4	11.5
210	151.7	192.3	11.2
225	111.3	159.2	11.5
240	85.5	122.3	11.2
255	66.0	67.8	11.6
270	61.8	23.1	11.4
285	69.2	-20.9	11.2
300	88.7	-67.9	11.3
315	123.6	-112.3	11.3
330	159.0	-136.3	10.5
345	193.2	-148.1	10.6
360	228.9	-151.6	10.7

average turn rate = 2.31 Deg/Sec

Time to turn
90 0 min 36 sec
180 1 min 17 sec
270 2 min 2 sec
360 2 min 45 sec

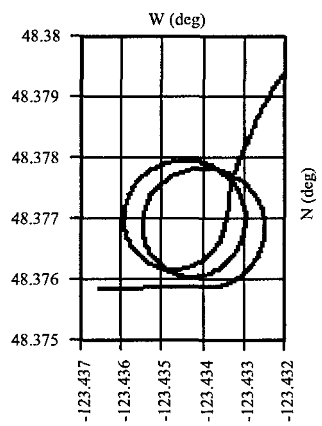
HMCS THUNDER TURNING CHARACTERISTICS

AT 12 KNOTS PORT 15 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	6.2	0.0	11.0
15	153.7	-6.0	8.2
30	157.9	-6.9	7.5
45	162.4	-7.4	8.2
60	167.5	-7.3	9.1
75	172.8	-7.0	9.4
90	178.3	-6.4	9.8
105	183.8	-5.3	9.9
120	189.2	-3.8	10.1
135	194.8	-2.2	10.2
150	200.3	-0.3	10.4
165	205.7	2.0	10.5
180	211.0	4.7	10.6
195	216.3	7.6	10.8
210	221.3	10.5	10.2
225	226.1	14.1	10.6
240	230.9	17.6	10.6
255	235.5	21.4	10.5
270	239.8	25.4	10.4
285	244.0	29.8	10.8
300	247.9	34.0	10.2
315	251.5	38.6	10.4
330	255.0	43.3	10.4
345	258.2	48.4	10.6
360	260.8	53.3	9.9

average turn rate = (2.44) Deg/Sec

Time to turn
90 0 min 30 sec
180 0 min 36 sec
270 0 min 42 sec
360 0 min 48 sec



HMCS THUNDER TURNING CHARACTERISTICS

AT 12 KNOTS PORT 15 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	260.8	53.3	9.9
15	263.6	58.4	10.3
30	266.2	63.7	10.6
45	268.5	69.3	10.6
60	270.2	74.8	10.4
75	271.5	80.4	10.2
90	275.2	109.9	10.1
105	269.7	145.4	10.6
120	253.5	177.4	10.7
135	237.4	195.1	10.6
150	213.0	213.0	10.7
165	179.3	226.2	10.7
180	149.3	229.4	10.6
195	107.4	221.7	10.8
210	86.2	210.7	10.6
225	63.6	193.3	9.9
240	46.4	171.0	9.9
255	35.1	139.8	10.2
270	33.2	111.7	10.1
285	38.1	84.7	10.0
300	53.0	56.1	9.2
315	67.4	39.9	9.4
330	89.4	24.2	9.5
345	114.1	16.3	9.6
360	135.1	15.4	9.2

average turn rate = (3.22) Deg/Sec

Time to turn
90 0 min 10 sec
180 0 min 42 sec
270 1 min 14 sec
360 1 min 43 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 12 KNOTS PORT 15 DEGREES OF HELM (circle 3)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	135.1	15.4	9.2
15	156.5	19.4	9.7
30	181.4	30.4	10.0
45	206.8	51.7	10.1
60	222.9	75.3	10.3
75	234.8	108.1	10.2
90	236.3	143.5	10.6
105	230.8	166.3	10.5
120	218.7	192.7	10.1
135	195.1	219.9	11.0
150	169.5	236.6	11.1
165	140.6	245.8	10.8
180	116.6	247.8	10.8
195	81.1	242.8	10.7
210	48.5	227.6	10.6
225	22.3	204.5	10.5
240	9.8	185.2	10.4
255	-1.6	152.6	10.2
270	-3.1	124.6	9.8
285	1.2	103.2	9.6
300	12.6	78.7	9.4
315	29.5	57.9	9.4
330	51.0	42.0	9.5
345	76.2	33.6	9.4
360	97.3	31.9	9.4

average turn rate = 2.89 Deg/Sec

Time to turn

90	0 min 32 sec
180	1 min 1 sec
270	1 min 34 sec
360	2 min 2 sec

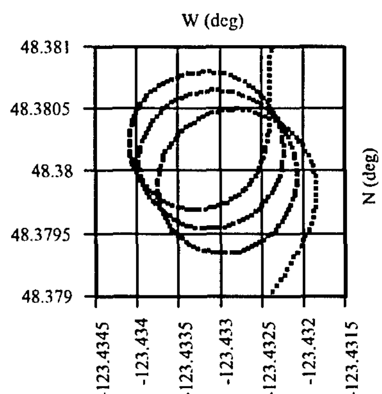
HMCS THUNDER TURNING CHARACTERISTICS

AT 12 KNOTS PORT 30 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	6.5	0.0	11.6
15	93.8	3.4	10.4
30	119.7	16.1	10.5
45	132.1	27.1	9.7
60	144.4	45.0	9.7
75	151.1	65.1	9.5
90	152.3	86.0	9.3
105	146.5	113.0	10.0
120	141.6	122.5	8.9
135	125.0	140.2	8.8
150	108.0	150.7	9.2
165	94.1	155.5	8.8
180	74.5	157.5	8.7
195	50.6	152.7	8.8
210	37.6	146.1	9.0
225	23.4	134.1	8.3
240	12.9	119.2	8.1
255	8.3	106.8	7.7
270	7.1	84.4	8.0
285	10.2	71.1	8.1
300	18.2	55.3	7.9
315	30.4	42.1	8.1
330	42.1	34.4	8.4
345	60.2	28.0	8.8
360	84.2	25.9	8.9

average turn rate = 3.09 Deg/Sec

Time to turn
90 0 min 35 sec
180 0 min 58 sec
270 1 min 22 sec
360 1 min 45 sec



HMCS THUNDER TURNING CHARACTERISTICS

AT 12 KNOTS PORT 30 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	84.2	25.9	8.9
15	103.6	30.1	9.2
30	121.9	38.9	9.4
45	138.1	52.4	9.4
60	152.9	73.9	9.1
75	159.1	93.7	9.3
90	160.0	109.4	9.4
105	154.5	135.0	9.5
120	144.9	153.1	9.1
135	134.5	164.0	9.0
150	114.0	177.9	8.8
165	95.2	183.4	8.5
180	80.5	183.8	8.8
195	66.1	181.2	8.9
210	49.0	173.2	8.3
225	38.3	164.5	7.9
240	28.0	149.8	8.1
255	23.3	136.7	8.3
270	20.6	114.3	8.2
285	23.8	96.4	7.9
300	31.6	80.2	7.7
315	41.1	69.3	8.8
330	61.2	55.6	8.4
345	74.7	50.9	8.4
360	98.4	48.7	8.5

average turn rate = 3.80 Deg/Sec

Time to turn

90	0 min 24 sec
180	0 min 48 sec
270	1 min 10 sec
360	1 min 34 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 12 KNOTS PORT 30 DEGREES OF HELM (circle 3)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	98.4	48.7	8.5
15	117.4	52.6	8.5
30	135.1	60.9	8.7
45	153.7	77.9	8.9
60	161.7	90.8	9.3
75	168.7	110.0	8.9
90	170.2	130.5	9.0
105	166.2	150.9	9.1
120	156.6	169.0	9.1
135	146.6	180.9	9.2
150	126.1	195.1	8.8
165	112.3	200.1	8.6
180	93.4	201.8	8.5
195	74.7	198.1	8.5
210	61.6	191.1	8.7
225	47.9	179.9	7.5
240	37.9	165.5	7.7
255	32.2	148.9	7.7
270	31.4	131.0	8.0
285	35.3	113.5	7.8
300	43.6	97.7	7.9
315	52.9	88.0	8.1
330	68.1	78.0	8.4
345	81.3	73.4	8.6
360	100.7	71.3	8.7

average turn rate = 3.84 Deg/Sec

Time to turn

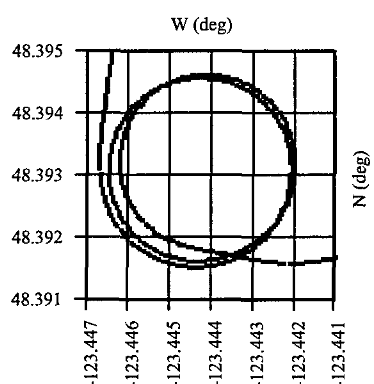
90	0 min 24 sec
180	0 min 47 sec
270	1 min 10 sec
360	1 min 32 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS PORT 7 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	8.2	0.0	14.6
15	266.2	13.2	14.2
30	324.6	38.0	14.0
45	367.0	73.1	14.0
60	389.1	104.7	13.6
75	408.6	156.9	14.3
90	412.8	204.2	14.0
105	404.3	250.6	14.3
120	384.3	293.8	14.2
135	354.0	330.2	14.4
150	321.9	354.1	14.3
165	269.8	375.1	14.5
180	223.1	379.9	13.8
195	167.1	371.8	14.5
210	124.1	351.7	13.9
225	88.2	321.5	13.7
240	61.9	283.2	13.8
255	49.1	246.9	13.8
270	44.1	193.5	13.7
285	50.2	155.3	13.8
300	76.3	98.9	14.0
315	97.1	75.3	14.0
330	133.7	47.1	13.7
345	184.2	29.0	13.8
360	230.9	25.6	14.1

average turn rate = 1.78 Deg/Sec



Time to turn
90 1 min 6 sec
180 1 min 42 sec
270 2 min 19 sec
360 2 min 55 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS PORT 7 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	230.9	25.6	14.1
15	277.6	34.0	14.2
30	320.4	53.9	14.1
45	356.5	84.1	13.8
60	382.8	122.7	14.0
75	397.9	167.2	14.1
90	400.3	214.1	14.2
105	392.3	252.7	14.1
120	372.6	296.1	14.2
135	342.5	333.4	14.0
150	297.8	366.0	14.0
165	260.6	379.8	14.0
180	204.6	385.7	14.5
195	157.1	376.7	14.2
210	114.2	356.4	14.1
225	78.4	326.5	13.7
240	56.3	294.9	14.0
255	40.1	252.3	13.6
270	36.0	206.7	13.5
285	45.5	153.8	13.9
300	58.9	126.2	13.4
315	92.4	84.9	13.4
330	130.3	60.1	13.4
345	166.2	48.3	13.1
360	211.7	43.9	13.8

average turn rate = 2.51 Deg/Sec

Time to turn

90	0 min 36 sec
180	1 min 12 sec
270	1 min 47 sec
360	2 min 22 sec

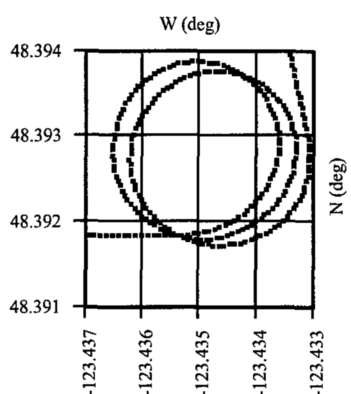
HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS PORT 15 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	8.4	0.0	14.9
15	84.6	-2.3	14.9
30	93.1	-2.9	15.1
45	101.5	-3.5	15.1
60	109.9	-4.3	14.9
75	118.2	-4.9	14.9
90	126.5	-5.5	14.9
105	134.5	-5.9	14.1
120	142.7	-6.3	14.6
135	150.8	-6.5	14.4
150	158.9	-6.5	14.4
165	166.8	-6.1	14.2
180	174.7	-5.6	14.0
195	182.6	-4.6	14.1
210	190.4	-3.6	14.1
225	198.2	-1.9	14.1
240	205.7	0.2	13.9
255	213.1	2.6	13.9
270	220.4	5.3	13.8
285	227.7	8.2	13.9
300	234.6	12.0	14.0
315	241.3	16.1	14.0
330	247.9	20.5	14.0
345	254.0	25.2	13.9
360	259.9	30.4	14.0

average turn rate = (0.93) Deg/Sec

Time to turn
90 0 min 15 sec
180 0 min 21 sec
270 0 min 27 sec
360 0 min 33 sec



HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS PORT 15 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	259.9	30.4	14.0
15	265.6	35.9	13.9
30	270.7	41.7	13.8
45	275.8	47.8	14.0
60	288.2	67.3	13.6
75	299.9	102.9	13.2
90	301.8	124.9	13.0
105	295.7	160.7	13.0
120	279.3	193.0	12.8
135	259.4	214.1	13.2
150	235.1	229.3	12.8
165	207.7	238.2	12.8
180	179.2	239.5	12.7
195	143.6	231.5	13.2
210	123.9	221.1	13.2
225	96.0	196.5	13.3
240	79.9	171.2	13.1
255	68.0	135.4	13.6
270	66.4	105.5	13.4
285	71.5	75.8	13.3
300	88.4	41.8	13.7
315	109.1	18.7	13.9
330	134.5	1.2	13.7
345	170.5	-11.5	13.8
360	200.6	-13.6	13.5

average turn rate = 4.73 Deg/Sec

Time to turn

90	0 min 14 sec
180	0 min 40 sec
270	1 min 6 sec
360	1 min 32 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS PORT 15 DEGREES OF HELM (circle 3)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	200.6	-13.6	13.5
15	237.9	-5.9	13.7
30	265.2	7.8	13.6
45	288.7	27.2	13.6
60	314.3	62.9	12.7
75	321.6	83.3	12.5
90	325.4	118.1	12.3
105	320.6	145.7	12.5
120	305.1	177.0	12.4
135	286.6	197.8	12.7
150	263.7	213.1	12.3
165	237.0	222.3	12.6
180	208.7	224.7	13.0
195	173.2	217.3	13.0
210	140.4	200.0	13.4
225	124.0	184.7	13.3
240	102.6	154.5	13.2
255	92.5	126.7	12.9
270	89.9	97.6	13.0
285	96.9	61.3	13.4
300	110.4	34.7	13.4
315	135.4	6.6	13.4
330	160.3	-10.0	13.3
345	195.9	-21.7	13.3
360	225.8	-23.4	13.5

average turn rate = 3.42 Deg/Sec

Time to turn

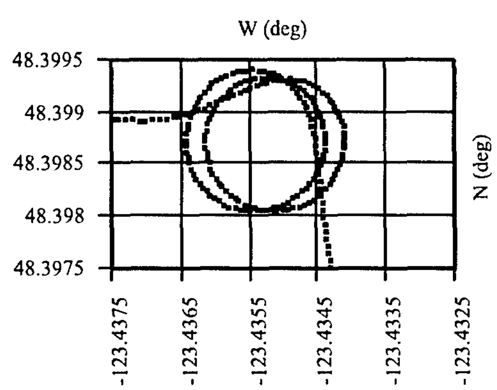
90	0 min 27 sec
180	0 min 52 sec
270	1 min 18 sec
360	1 min 45 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS PORT 30 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	8.4	0.0	14.9
15	178.4	4.2	13.0
30	196.9	14.5	12.1
45	211.2	28.4	11.6
60	218.2	39.5	11.5
75	223.9	57.8	11.3
90	224.3	70.4	11.2
105	220.2	95.6	11.7
120	210.6	112.9	12.0
135	196.9	128.4	12.5
150	179.9	139.6	12.0
165	154.1	148.1	12.3
180	133.9	149.2	11.7
195	114.3	145.2	11.8
210	96.6	136.8	11.5
225	77.5	119.4	11.6
240	67.3	103.1	11.4
255	61.5	84.6	11.5
270	60.3	65.0	11.5
285	65.4	39.1	11.8
300	74.7	21.2	12.3
315	91.9	1.7	11.4
330	108.3	-8.3	11.4
345	126.5	-14.3	11.6
360	145.5	-15.8	11.3

average turn rate = 3.33 Deg/Sec



Time to turn
90 0 min 35 sec
180 0 min 55 sec
270 1 min 14 sec
360 1 min 34 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS PORT 30 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	145.5	-15.8	11.3
15	169.9	-10.5	11.1
30	186.6	-1.6	11.0
45	199.7	11.1	10.8
60	206.3	21.3	10.8
75	212.1	38.4	10.8
90	213.5	56.6	10.9
105	210.0	74.5	10.9
120	202.0	91.6	11.2
135	184.8	110.2	11.6
150	168.2	120.1	11.3
165	149.5	125.7	11.6
180	130.2	126.6	11.3
195	111.2	122.5	11.5
210	93.7	113.9	11.7
225	74.3	96.4	11.6
240	63.7	79.6	11.9
255	57.6	60.6	12.0
270	56.0	40.9	11.9
285	61.5	15.1	11.6
300	70.4	-2.1	11.7
315	83.2	-17.0	11.5
330	105.2	-31.1	11.4
345	123.6	-37.1	11.6
360	142.5	-38.3	11.3

average turn rate = 4.86 Deg/Sec

Time to turn

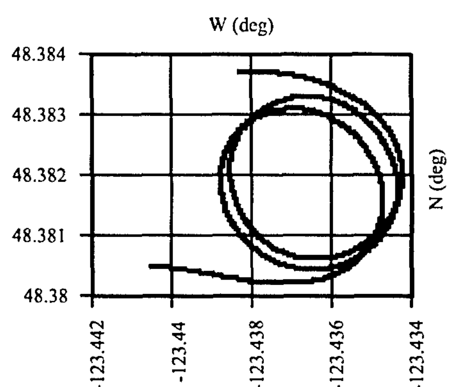
90	0 min 18 sec
180	0 min 37 sec
270	0 min 56 sec
360	1 min 16 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 9 KNOTS STARBOARD 7 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	4.6	0.0	8.1
15	81.7	9.1	8.2
30	189.5	53.6	8.2
45	232.7	88.3	8.1
60	263.2	128.8	8.6
75	278.5	167.8	8.6
90	282.9	204.9	8.7
105	277.2	245.9	7.9
120	258.5	289.8	8.5
135	237.4	316.5	8.7
150	189.3	353.9	8.8
165	152.1	368.1	8.9
180	102.8	372.9	8.9
195	59.7	366.7	8.8
210	19.3	349.2	8.6
225	-17.9	319.4	8.2
240	-42.7	285.1	8.6
255	-62.7	231.5	8.5
270	-66.6	194.6	8.3
285	-60.6	153.4	7.9
300	-45.8	120.5	8.0
315	-20.3	89.9	7.8
330	5.3	71.2	8.0
345	46.9	54.6	8.1
360	95.3	48.1	7.9

average turn rate =1.29 Deg/Sec



Time to turn
90 1 min 24 sec
180 2 min 20 sec
270 3 min 17 sec
360 4 min 11 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 9 KNOTS STARBOARD 7 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	95.3	48.1	7.9
15	139.2	54.2	8.0
30	171.5	68.2	7.8
45	217.4	103.7	8.0
60	255.1	155.8	8.1
75	269.7	195.0	8.6
90	274.8	231.9	8.3
105	270.5	268.4	8.2
120	253.1	311.1	8.4
135	220.2	350.4	8.4
150	189.1	372.5	8.4
165	149.4	388.6	8.4
180	106.7	394.9	8.6
195	49.3	387.1	8.8
210	4.6	367.7	8.6
225	-39.8	332.0	8.7
240	-65.0	297.9	8.5
255	-80.2	258.1	8.5
270	-85.0	221.6	7.9
285	-80.5	175.7	8.2
300	-62.5	134.7	8.1
315	-42.4	111.0	7.9
330	-13.5	90.7	7.6
345	19.1	78.3	7.6
360	63.6	72.2	7.9

average turn rate = 1.52 Deg/Sec

Time to turn
90 1 min 2 sec
180 1 min 57 sec
270 2 min 57 sec
360 3 min 50 sec

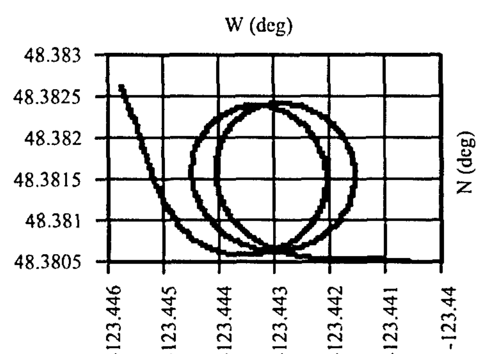
HMCS THUNDER TURNING CHARACTERISTICS

AT 9 KNOTS STARBOARD 15 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	172.2	22.2	8.8
15	176.5	24.9	9.0
30	180.3	27.8	8.6
45	199.1	44.1	9.1
60	219.3	71.7	8.7
75	229.5	97.7	7.8
90	233.4	128.4	7.6
105	227.4	162.4	7.8
120	218.4	181.2	7.1
135	203.1	200.1	7.1
150	186.4	211.9	7.2
165	163.5	221.2	7.3
180	134.0	224.0	7.5
195	105.1	218.6	7.4
210	78.7	205.8	7.6
225	59.2	190.2	7.7
240	43.7	169.5	8.0
255	33.0	141.4	7.7
270	30.9	111.7	7.6
285	35.5	85.9	8.0
300	49.1	57.8	8.2
315	63.4	41.1	7.7
330	87.9	22.7	7.8
345	120.8	9.8	8.3
360	148.0	6.9	8.2

average turn rate = 2.81 Deg/Sec

Time to turn
90 0 min 27 sec
180 1 min 4 sec
270 1 min 44 sec
360 2 min 23 sec



HMCS THUNDER TURNING CHARACTERISTICS

AT 9 KNOTS STARBOARD 15 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	148.0	6.9	8.2
15	178.7	11.2	7.8
30	210.9	25.2	7.8
45	231.6	41.6	7.8
60	253.2	70.0	7.7
75	265.8	102.2	7.7
90	268.4	126.6	7.2
105	265.7	147.1	7.3
120	252.1	177.2	7.5
135	236.2	195.7	7.2
150	216.0	209.5	7.1
165	193.0	217.6	7.3
180	172.3	219.8	7.4
195	147.9	216.5	7.3
210	121.1	204.4	7.6
225	99.2	185.5	7.1
240	85.2	164.8	7.6
255	73.2	132.3	7.7
270	70.8	106.4	7.6
285	76.2	77.0	7.6
300	89.7	50.1	7.6
315	107.1	30.7	7.6
330	133.0	13.0	8.0
345	162.7	2.3	8.0
360	193.8	-1.0	7.9

average turn rate = 2.22 Deg/Sec

Time to turn

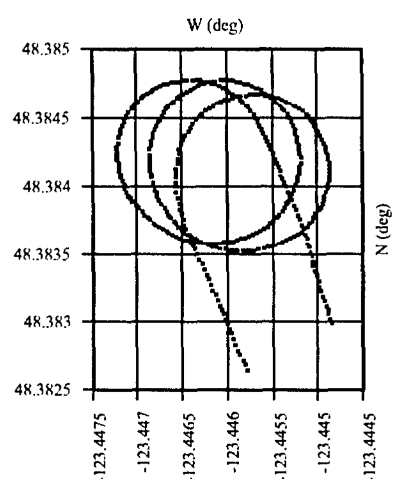
90	0 min 43 sec
180	1 min 19 sec
270	1 min 59 sec
360	2 min 40 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 9 KNOTS STARBOARD 30 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	4.9	0.0	8.7
15	77.1	7.7	8.3
30	100.7	20.1	7.7
45	110.4	28.7	7.8
60	124.2	48.9	7.2
75	128.8	63.8	6.8
90	129.7	83.0	6.6
105	125.6	100.9	6.5
120	119.1	114.2	6.5
135	107.3	128.0	6.3
150	92.3	138.0	6.4
165	75.8	143.5	6.2
180	58.4	145.0	6.2
195	41.7	142.1	5.8
210	26.2	135.1	5.8
225	15.5	126.1	6.2
240	1.0	105.6	6.5
255	-3.9	91.9	6.4
270	-5.9	69.7	6.5
285	-2.5	47.4	6.8
300	4.8	30.3	6.7
315	21.0	10.1	6.5
330	38.5	-1.9	6.2
345	55.2	-7.7	6.5
360	79.9	-10.6	6.2

average turn rate = 2.55 Deg/Sec



Time to turn
90 0 min 40 sec
180 1 min 9 sec
270 1 min 40 sec
360 2 min 16 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 9 KNOTS STARBOARD 30 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	79.9	-10.6	6.2
15	100.8	-7.2	6.2
30	120.4	2.3	6.2
45	133.8	14.0	6.6
60	143.8	28.3	6.3
75	149.7	44.7	6.2
90	151.1	62.2	6.3
105	148.1	79.2	6.0
120	140.4	95.2	6.6
135	129.5	108.0	5.9
150	115.2	117.4	6.0
165	98.8	123.3	6.3
180	77.3	124.4	6.4
195	63.2	121.5	6.2
210	46.6	114.0	6.9
225	29.8	99.6	6.7
240	19.3	83.9	6.8
255	12.6	62.7	6.6
270	11.9	48.0	6.5
285	16.2	22.9	6.5
300	24.3	6.4	6.5
315	40.4	-12.5	6.3
330	57.6	-24.5	6.2
345	81.1	-32.5	6.2
360	102.1	-33.5	6.2

average turn rate = 2.83 Deg/Sec

Time to turn

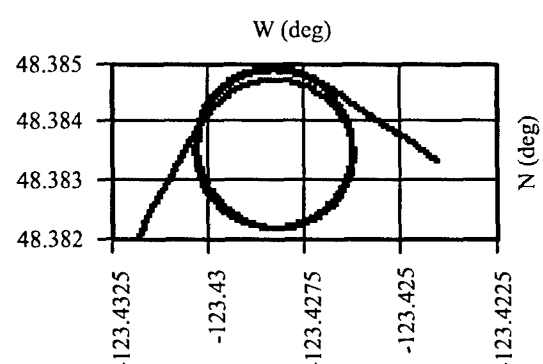
90	0 min 32 sec
180	1 min 3 sec
270	1 min 33 sec
360	2 min 11 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 12 KNOTS STARBOARD 7 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	6.4	0.0	11.4
15	230.4	7.4	11.8
30	271.3	27.1	11.1
45	300.4	51.5	11.3
60	326.1	87.2	11.3
75	344.2	134.0	11.3
90	347.6	171.0	11.0
105	339.4	219.9	11.1
120	320.0	258.9	11.2
135	295.9	286.7	10.7
150	260.1	310.6	11.2
165	225.1	321.8	11.1
180	187.8	323.5	11.4
195	150.4	317.0	11.1
210	115.5	301.3	11.4
225	80.2	273.4	11.3
240	57.7	242.2	11.5
255	42.0	198.8	11.7
270	38.1	153.4	12.0
285	46.5	108.2	11.7
300	63.0	72.3	11.4
315	91.0	36.4	11.5
330	127.5	9.8	11.6
345	169.9	-6.1	11.3
360	208.1	-9.8	11.1

average turn rate = 1.67 Deg/Sec



Time to turn
90 1 min 9 sec
180 1 min 49 sec
270 2 min 28 sec
360 3 min 8 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 12 KNOTS STARBOARD 7 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	208.1	-9.8	11.1
15	245.5	-4.0	11.3
30	285.6	13.9	11.3
45	314.6	38.2	11.3
60	340.9	74.0	11.3
75	360.6	127.1	11.0
90	363.6	164.5	11.4
105	358.6	195.1	11.1
120	337.0	242.7	11.4
135	305.5	279.9	10.3
150	270.4	303.4	10.9
165	241.1	313.8	11.0
180	196.9	318.2	11.4
195	159.0	313.2	11.3
210	105.7	289.7	11.7
225	76.3	263.9	11.7
240	52.7	232.1	11.7
255	36.1	189.6	11.4
270	32.0	144.6	11.4
285	38.7	106.4	11.7
300	61.6	59.6	11.6
315	83.0	35.4	11.5
330	114.6	11.6	11.6
345	150.6	-5.0	11.7
360	208.6	-12.8	11.6

average turn rate = 3.33 Deg/Sec

Time to turn

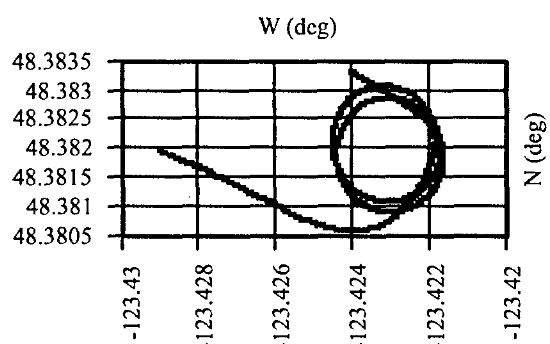
90	0 min 41 sec
180	1 min 21 sec
270	2 min 2 sec
360	2 min 42 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 12 KNOTS STARBOARD 15 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	6.1	0.0	10.8
15	189.2	5.2	11.1
30	217.9	18.3	11.4
45	242.2	38.4	11.4
60	259.8	64.7	10.8
75	268.0	87.9	10.7
90	270.3	110.9	10.0
105	264.7	145.0	10.3
120	251.9	171.6	10.7
135	232.9	194.2	10.8
150	208.4	210.9	10.6
165	180.5	220.5	10.5
180	150.9	222.9	10.4
195	122.3	217.3	10.4
210	95.1	204.2	10.7
225	76.6	188.6	11.1
240	54.5	158.6	11.2
255	41.3	122.3	11.7
270	39.0	96.8	11.3
285	43.5	65.9	10.9
300	59.8	32.9	10.8
315	76.0	15.2	10.7
330	100.3	-1.4	10.6
345	122.4	-9.1	10.6
360	151.5	-11.5	10.2

average turn rate = 2.21 Deg/Sec



Time to turn
90 0 min 53 sec
180 1 min 24 sec
270 1 min 54 sec
360 2 min 23 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 12 KNOTS STARBOARD 15 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	151.5	-11.5	10.2
15	180.7	-5.5	10.8
30	207.3	8.1	10.6
45	225.5	24.0	11.0
60	242.4	48.1	10.5
75	252.1	76.1	10.7
90	253.9	105.0	10.1
105	249.9	127.9	10.3
120	235.0	159.7	10.7
135	215.6	181.2	10.5
150	195.8	194.4	10.6
165	167.3	204.6	10.9
180	137.0	207.0	10.6
195	106.4	201.6	11.1
210	77.8	188.7	11.4
225	48.5	164.6	11.4
240	30.4	138.3	11.2
255	19.7	109.1	11.1
270	17.3	77.9	11.4
285	22.6	47.4	10.7
300	35.8	20.0	10.5
315	55.3	-1.9	10.2
330	74.6	-14.9	10.3
345	102.1	-24.1	10.5
360	131.7	-25.8	10.8

average turn rate = 3.07 Deg/Sec

Time to turn

90	0 min 29 sec
180	0 min 58 sec
270	1 min 29 sec
360	1 min 58 sec

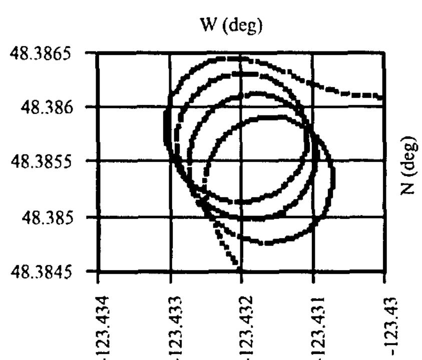
HMCS THUNDER TURNING CHARACTERISTICS

AT 12 KNOTS STARBOARD 30 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	6.8	0.0	12.2
15	74.6	5.5	12.0
30	103.7	18.9	11.2
45	116.6	31.0	10.2
60	129.5	49.8	10.0
75	135.7	70.8	9.7
90	136.1	86.7	9.6
105	131.7	107.1	9.4
120	122.2	125.1	9.1
135	111.9	136.4	9.2
150	99.2	144.7	9.0
165	80.5	151.6	8.7
180	60.1	152.5	9.3
195	40.7	148.2	8.8
210	27.4	141.2	9.0
225	12.6	128.0	9.0
240	2.1	110.7	9.1
255	-2.5	96.6	8.5
270	-3.4	81.4	9.2
285	0.2	61.1	9.1
300	12.3	37.7	9.4
315	23.0	25.6	9.7
330	45.5	10.1	9.8
345	66.2	2.7	9.8
360	93.2	0.9	9.8

average turn rate = 3.45 Deg/Sec

Time to turn
90 0 min 30 sec
180 0 min 52 sec
270 1 min 13 sec
360 1 min 39 sec



HMCS THUNDER TURNING CHARACTERISTICS

AT 12 KNOTS STARBOARD 30 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	93.2	0.9	9.8
15	109.1	4.0	9.7
30	133.5	15.0	9.0
45	148.8	29.0	9.1
60	157.8	42.0	9.6
75	166.2	66.7	9.3
90	166.7	87.3	9.3
105	161.7	107.2	9.3
120	154.5	120.9	9.1
135	141.1	135.8	9.0
150	128.2	144.0	9.0
165	109.0	150.1	9.0
180	94.0	151.1	9.0
195	79.6	148.3	9.0
210	61.5	140.3	8.3
225	46.7	127.4	9.0
240	38.3	115.0	8.7
255	30.8	96.3	8.7
270	29.2	71.2	8.9
285	34.2	51.6	9.0
300	41.2	37.9	9.3
315	54.7	22.0	9.3
330	72.3	10.3	9.6
345	97.5	1.3	9.3
360	119.0	0.1	9.6

average turn rate = 3.91 Deg/Sec

Time to turn

90	0 min 24 sec
180	0 min 45 sec
270	1 min 8 sec
360	1 min 32 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 12 KNOTS STARBOARD 30 DEGREES OF HELM (circle 3)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	119.0	0.1	9.6
15	139.7	4.5	9.6
30	159.1	14.0	9.7
45	171.4	24.6	9.7
60	186.7	46.2	9.6
75	192.9	65.9	9.1
90	194.1	81.4	9.4
105	190.3	101.7	9.1
120	181.3	120.7	9.3
135	167.5	136.4	9.2
150	154.4	144.9	9.2
165	134.9	151.8	9.2
180	115.0	152.7	8.5
195	100.3	149.3	8.9
210	82.3	140.6	9.0
225	67.7	126.7	9.2
240	57.0	109.7	9.2
255	51.7	90.4	8.8
270	51.4	75.1	8.8
285	55.8	54.7	9.4
300	65.5	36.0	9.3
315	79.6	20.2	9.4
330	102.7	6.1	9.8
345	118.0	1.4	9.8
360	144.6	-0.5	9.6

average turn rate = 3.92 Deg/Sec

Time to turn

90	0 min 23 sec
180	0 min 46 sec
270	1 min 8 sec
360	1 min 33 sec

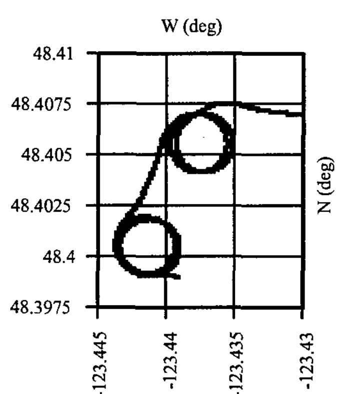
HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS STARBOARD 7 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	8.0	0.0	14.1
15	248.6	19.2	14.6
30	300.6	43.4	14.4
45	331.6	69.6	14.5
60	358.5	109.9	14.6
75	371.5	147.8	14.3
90	375.1	188.5	14.1
105	366.0	236.3	14.7
120	348.1	272.8	14.6
135	321.2	303.8	14.2
150	288.1	327.4	14.5
165	241.9	344.2	14.8
180	192.3	347.2	15.0
195	150.6	339.7	15.1
210	96.8	315.1	15.2
225	65.1	287.4	15.0
240	37.5	246.0	14.5
255	24.8	206.6	14.9
270	21.7	165.9	14.5
285	30.9	118.5	14.3
300	48.1	82.7	14.0
315	79.0	46.4	14.2
330	111.9	24.4	13.9
345	149.3	10.7	14.2
360	188.7	6.6	13.9

average turn rate = 1.98 Deg/Sec

Time to turn
90 0 min 59 sec
180 1 min 32 sec
270 2 min 5 sec
360 2 min 37 sec



HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS STARBOARD 7 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	188.7	6.6	13.9
15	243.0	16.6	13.9
30	278.2	34.6	14.0
45	308.8	60.4	14.3
60	335.2	100.1	14.1
75	347.7	138.1	14.2
90	350.8	178.4	14.5
105	343.1	217.1	14.2
120	325.7	254.0	14.9
135	292.6	291.8	15.2
150	264.9	311.1	15.3
165	217.7	329.2	14.1
180	158.5	335.1	15.0
195	117.3	326.8	14.8
210	72.8	305.1	14.6
225	41.8	277.6	14.8
240	14.4	236.8	14.6
255	1.2	198.2	14.3
270	-2.2	158.2	14.5
285	7.0	110.8	14.4
300	25.0	74.3	14.7
315	50.8	43.3	14.3
330	83.0	19.9	14.1
345	128.2	3.4	14.1
360	183.3	-1.2	14.1

average turn rate = 2.73 Deg/Sec

Time to turn

90	0 min 33 sec
180	1 min 6 sec
270	1 min 38 sec
360	2 min 12 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS STARBOARD 7 DEGREES OF HELM (circle 3)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	183.3	-1.2	14.1
15	238.9	7.3	14.4
30	290.5	30.7	14.6
45	333.9	67.7	14.6
60	361.1	108.3	14.6
75	374.5	146.5	14.3
90	377.9	179.0	15.5
105	370.1	228.4	14.9
120	339.2	295.2	14.4
135	161.8	672.4	14.4
150	159.9	680.4	14.6
165	158.0	688.2	14.3
180	156.1	696.2	14.6
195	154.2	704.0	14.3
210	152.5	711.9	14.2
225	151.0	719.9	14.5
240	149.7	727.6	14.1
255	148.3	735.6	14.4
270	147.0	743.6	14.4
285	145.2	751.6	14.6
300	143.5	759.7	14.6
315	141.8	767.7	14.5
330	140.1	775.5	14.2
345	138.2	783.5	14.6
360	136.3	791.3	14.3

average turn rate = 2.43 Deg/Sec

Time to turn

90	0 min 36 sec
180	1 min 45 sec
270	1 min 51 sec
360	1 min 57 sec

Note that turning circle at 15 knots and starboard 7 was aborted and restarted. However, the advance and transfer are referenced from the initial command.

HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS STARBOARD 7 DEGREES OF HELM (circle 4)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	136.3	791.3	14.3
15	134.3	799.3	14.7
30	132.1	807.2	14.4
45	129.8	814.8	14.2
60	127.1	822.7	14.8
75	124.1	830.5	15.0
90	121.0	838.2	14.7
105	117.8	845.9	14.8
120	102.3	875.9	15.2
135	64.5	922.6	15.3
150	22.7	952.7	15.3
165	-34.1	973.5	15.5
180	-76.0	976.7	14.9
195	-125.4	967.4	15.0
210	-170.8	945.8	14.8
225	-208.4	913.5	14.4
240	-232.2	879.6	14.9
255	-248.9	832.7	15.0
270	-251.3	790.4	14.9
285	-241.4	742.7	13.9
300	-220.4	700.4	14.0
315	-194.9	671.5	13.6
330	-163.0	649.3	13.7
345	-119.4	633.3	13.9
360	-81.0	629.7	13.7

average turn rate = 6.84 Deg/Sec

Time to turn

90	0 min 6 sec
180	0 min 36 sec
270	1 min 10 sec
360	1 min 43 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS STARBOARD 7 DEGREES OF HELM (circle 5)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	-81.0	629.7	13.7
15	-35.3	637.7	13.8
30	6.4	658.2	13.8
45	36.2	683.2	14.0
60	67.8	728.6	14.1
75	80.9	766.1	14.2
90	84.9	806.3	14.5
105	76.3	854.9	14.7
120	54.0	899.5	14.7
135	26.5	931.2	14.9
150	-14.9	960.3	14.9
165	-62.2	977.4	14.8
180	-111.9	980.7	14.7
195	-153.1	973.1	15.0
210	-191.8	956.0	15.2
225	-238.1	918.9	14.9
240	-262.5	884.6	14.9
255	-280.4	838.0	15.1
270	-284.7	789.0	14.5
285	-275.1	741.4	14.0
300	-257.2	706.0	14.0
315	-231.4	675.8	14.3
330	-192.9	649.0	13.9
345	-156.4	635.6	13.9
360	-117.5	631.6	14.1

average turn rate = 2.64 Deg/Sec

Time to turn

90	0 min 34 sec
180	1 min 9 sec
270	1 min 43 sec
360	2 min 15 sec

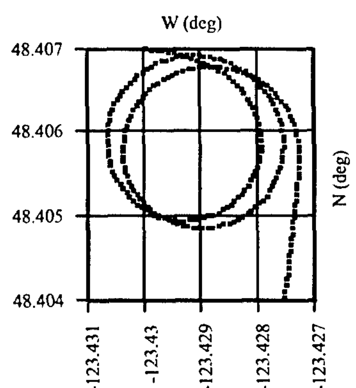
HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS STARBOARD 15 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	8.6	0.0	15.4
15	76.4	9.9	15.4
30	115.0	27.9	15.0
45	140.5	49.3	14.7
60	163.6	82.8	14.3
75	170.6	105.9	14.3
90	172.9	144.8	13.6
105	168.1	166.7	13.1
120	151.9	200.2	13.3
135	131.9	221.4	13.0
150	113.8	233.2	12.7
165	86.7	242.9	12.9
180	58.2	245.1	12.5
195	29.8	238.9	13.0
210	9.8	229.4	13.4
225	-18.3	205.5	13.3
240	-34.3	180.4	13.3
255	-43.8	152.0	13.1
270	-45.6	121.6	13.8
285	-37.6	83.8	14.0
300	-22.8	56.3	14.1
315	-1.7	33.5	13.8
330	24.5	16.4	13.8
345	54.2	6.1	13.9
360	85.3	3.9	14.0

average turn rate = 3.30 Deg/Sec

Time to turn
90 0 min 31 sec
180 0 min 54 sec
270 1 min 18 sec
360 1 min 43 sec



HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS STARBOARD 15 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	85.3	3.9	14.0
15	116.3	9.7	14.1
30	144.9	22.7	14.2
45	175.0	48.3	14.3
60	192.2	74.7	14.2
75	204.8	112.1	13.8
90	206.1	143.3	13.8
105	199.8	173.2	13.5
120	187.0	200.6	13.3
135	167.6	223.5	13.3
150	143.2	240.5	13.1
165	108.0	252.3	13.2
180	85.9	253.3	13.0
195	56.8	248.4	13.2
210	24.1	232.2	13.2
225	7.4	217.7	13.1
240	-9.5	193.7	13.2
255	-19.6	165.7	13.2
270	-21.6	136.0	13.4
285	-16.5	105.5	13.8
300	1.2	70.7	14.1
315	22.4	47.6	14.0
330	48.9	30.7	13.9
345	79.1	20.7	14.3
360	110.8	18.4	14.2

average turn rate = 3.59 Deg/Sec

Time to turn

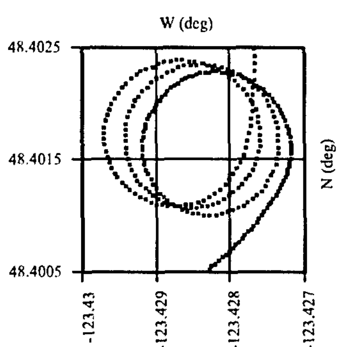
90	0 min 26 sec
180	0 min 50 sec
270	1 min 14 sec
360	1 min 39 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS STARBOARD 30 DEGREES OF HELM (circle 1)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	8.3	0.0	14.8
15	108.0	6.2	14.4
30	136.6	18.9	13.6
45	153.5	33.0	12.9
60	165.7	50.4	12.4
75	171.8	69.6	11.8
90	172.6	96.0	11.6
105	169.6	108.8	11.6
120	157.9	131.7	11.4
135	144.4	145.6	11.7
150	127.9	155.6	11.5
165	109.2	160.9	11.7
180	89.9	161.3	11.6
195	70.5	157.2	11.8
210	52.9	147.5	12.1
225	37.2	134.2	12.3
240	22.9	111.5	12.0
255	16.8	91.9	12.2
270	15.7	71.3	12.2
285	19.5	51.0	12.3
300	31.1	26.2	12.2
315	44.4	11.2	11.7
330	66.7	-3.2	11.7
345	79.4	-7.2	11.8
360	105.3	-8.5	11.2

average turn rate = 3.59 Deg/Sec



Time to turn
90 0 min 30 sec
180 0 min 48 sec
270 1 min 7 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS STARBOARD 30 DEGREES OF HELM (circle 2)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	105.3	-8.5	11.2
15	124.5	-3.9	11.6
30	141.9	5.4	11.8
45	156.1	18.1	11.3
60	167.0	33.9	11.3
75	174.2	57.8	11.1
90	174.4	70.1	10.8
105	171.2	88.4	11.0
120	163.0	105.2	11.2
135	151.1	119.6	10.9
150	129.8	133.4	11.4
165	111.5	138.2	11.3
180	92.0	138.8	11.6
195	72.8	134.3	11.6
210	55.4	125.2	11.6
225	40.6	111.9	11.8
240	26.2	89.0	12.0
255	20.3	69.6	12.1
270	19.3	49.1	12.2
285	23.1	28.8	12.3
300	32.0	10.2	12.2
315	49.4	-10.5	12.0
330	66.5	-21.4	12.0
345	85.4	-28.0	11.8
360	105.6	-29.8	11.9

average turn rate =4.87 Deg/Sec

Time to turn

90	0 min 18 sec
180	0 min 37 sec
270	0 min 56 sec
360	1 min 15 sec

HMCS THUNDER TURNING CHARACTERISTICS

AT 15 KNOTS STARBOARD 30 DEGREES OF HELM (circle 3)

AMOUNT OF TURN DEGREES	ADVANCE YARDS	TRANSFER YARDS	SPEED KNOTS
0	105.6	-29.8	11.9
15	131.6	-25.0	11.8
30	148.5	-15.9	11.3
45	162.9	-3.0	11.2
60	173.4	12.8	11.3
75	181.1	36.9	11.2
90	181.3	55.3	10.9
105	176.8	73.0	11.0
120	168.1	89.3	11.0
135	159.7	98.6	11.3
150	139.6	112.1	10.4
165	124.2	117.8	9.6
180	103.9	119.1	8.7
195	80.2	114.4	8.3
210	59.4	102.5	8.6
225	45.3	89.9	8.2
240	35.1	75.0	7.8
255	28.9	58.2	7.8
270	26.5	35.8	8.2
285	30.9	13.4	8.4
300	36.8	0.8	8.2
315	57.1	-24.0	8.1
330	73.0	-34.2	8.5
345	94.9	-41.9	8.1
360	112.7	-43.3	7.9

average turn rate = 4.25 Deg/Sec

Time to turn

90	0 min 20 sec
180	0 min 39 sec
270	1 min 6 sec
360	1 min 34 sec

Annex E

Solving For Model Parameters

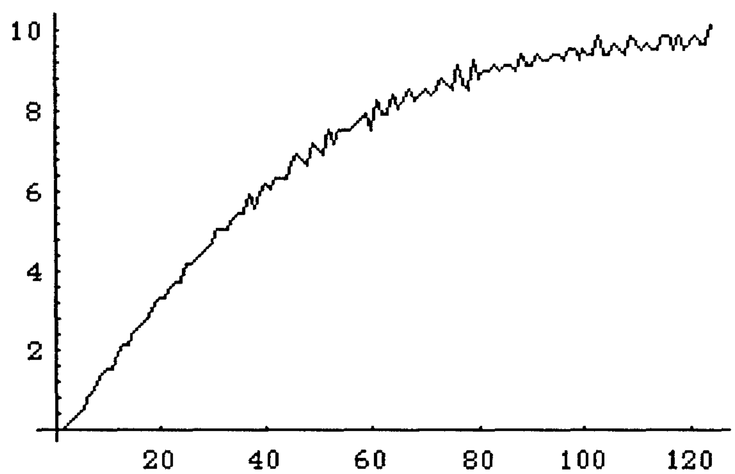
Time stamped speed data was derived from DGPS data and stored in computer files. The data was used to find the parameters that best fit a simple or quadratic lag function. The data is filtered using a fifth order polynomial function and then a set of points is located on the polynomial to solve for the model parameters. A correlation between the raw data and the model is found using the equation derived in Annex B.

The method for solving for the parameters is identical for both the simple, ideal, and quadratic lags, except that the quadratic lag has an additional parameter. The user chooses the desired model that best fits the data. The speed data file is retrieved, the number of data points is found, the data are zeroised, and the speed is plotted as a function of time. If the data file is for the deceleration phase, then a small routine inverts the data to appear like the acceleration data so that the same curve fitting routine can be applied to either data set.

The data are fitted by a fifth-order polynomial, and the first and second derivatives of the polynomial are found. The second derivative is needed only for the quadratic lag. The model parameters are solved from the differential expressions for the simple lag or quadratic lag at time intervals identified by the analyst. The solution is substituted into the given equation. All functions are reproduced and compared to the raw data, so that the analyst might estimate the correlation. Finally, the routine calculates the standard deviation and a correlation between the data and the model. Listed below is the Mathematica™ routine used to calculate the parameters.

Mathematica™ Routine

```
(* quadratic lag equation for imaginary roots *)
eqn = 9k4/b(1-E^(-a t/2)/Sqrt[1-a^2/4/b] Sin[Sqrt[b-a^2/4] t + ArcTan[Sqrt[4 b / a^2 -1]]])

(* quadratic lag equation for real roots *)
eqn = Simplify[Expand[k1 k4/(x y) ( 1 + y E^(-x t)/(x - y)- x E^(-y t)/(x - y) )]\
/. {x -> (a - Sqrt[a^2 - 4 b])/2,y -> (a + Sqrt[a^2 - 4 b])/2}]

(* simple lag equation for one root *)
eqn = k5 k4 / a ( 1 - E^(-a t) )

     -(a t)
(1 - E      ) k4 k5
--------------------
       a
(* Enter data for accelerating *)
data = ReadList["s10u", Number, RecordLists -> True];
n = Dimensions[data][[1]];
k5 = data[[n,2]]-data[[1,2]]
g1 = ListPlot[data, PlotRange -> Automatic, PlotJoined -> True]

10.1358
```




```
-Graphics-

(* Enter data for decccelerating *)
data = ReadList["f9d", Number, RecordLists -> True];
n = Dimensions[data][[1]];
di = data[[1,2]];
k5 = di-data[[n,2]]
Do[data[[i,2]] = di-data[[i,2]], {i,n}]
g1 = ListPlot[data, PlotRange -> Automatic, PlotJoined -> True]
```

```
Fit[data,Table[t^i,{i,1,5}], t];
c[t_] = %
g2 = Plot[c[t], {t,0,n}]
D[c[t],t];
dc[t_] = %;
Plot[dc[t], {t,0,n}];
D[dc[t],t];
ddc[t_] = %;
Plot[ddc[t], {t,0,n}];
```

$0.147544\, t + 0.00157993\, t^{2} - 0.0000520981\, t^{3} + 4.29378\ 10^{-7}\, t^{4} - 1.19337\ 10^{-9}\, t^{5}$

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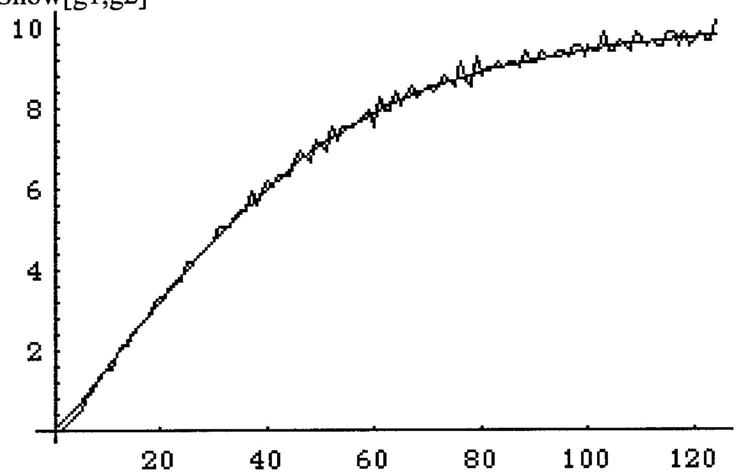
```
Show[g1,g2]
```



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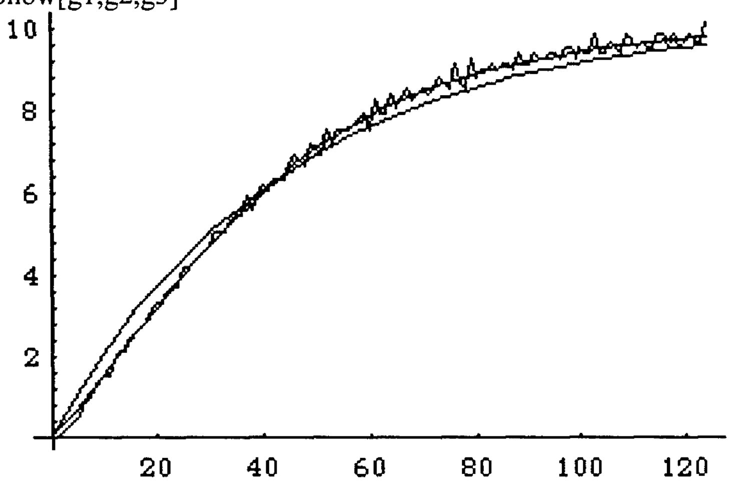
```
(* solution for quadratic lag *)
soln = Flatten[Solve[{N[ddc[i]] + a N[dc[i]] + b N[c[i]]  == k5 k4,\
                 N[ddc[j]] + a N[dc[j]] + b N[c[j]] == k5 k4,\
                 N[ddc[k]] + a N[dc[k]] + b N[c[k]] == k5 k4},\
                  {a,b,k4}]] /. {i -> 20, j -> 120, k -> 400}

(* natural frequency, damping ratio and gain *)
{w = Sqrt[b], z = a/2/Sqrt[b], k4} /. soln

(* solution for simple lag *)
soln = Flatten[Solve[{N[dc[i]] + a N[c[i]] == k5 k4,N[dc[j]] + a N[c[j]] == 10 k4},\
                {a,k4}]] /. {i -> 20, j -> 120}

{a -> 0.0231136, k4 -> 0.023253}
```

```
model[t_] = eqn /. soln
g3 = Plot[%, {t,0,n}];
(* incorporate these plots to compare to derivative plots of curve fit *)
                  -0.0231136 t
10.1969 (1 - E                  )

D[eqn,t] /. soln;
Plot[%, {t,0,n}];
D[D[eqn,t],t] /. soln;
Plot[%, {t,0,n}];

Show[g1,g2,g3]
```




```
-Graphics-

(* calculating the correlation between g1 and g3 *)
<<Statistics`DescriptiveStatistics`
ymean = Mean[Transpose[data][[2]]];
ssy = 0;
Do[ssy += (data[[i,2]]-ymean)^2, {i,n}]
ssy
1031.11
sserror = 0;
Do[sserror += (c[i]-model[i])^2, {i,n}]
Sqrt[sserror/(n-2)] (* deviation *)
r = Sqrt[1-sserror/ssy] (* correlation *)
0.322874
0.993814
```

Annex F

Correlation Calculation

The correlation (r) is derived from the sum of squared errors, SS_{error} (c.f. Gravetter & Wallnau, 1985). SS_{error} is a measure of the total deviation of the model from the data. It is found by forming a difference between a datum point (y_{data}) and the corresponding model point (y_{model}) at a given time, squaring it, and then summing the differences. The expression is as follows:

$$SS_{error} = \sum_{n} (y_{data} - y_{model})^2 \qquad \text{(F.1)}$$

where n is the number of data points. Note that the total variance (SS_y) in the data can be expressed as follows:

$$SS_y = \sum_{n} (y_{data} - \bar{y}_{data})^2 \quad \ldots \bar{y}_{data} = \sum_{n} \frac{y_{data}}{n} \qquad \text{(F.2)}$$

The unpredictable, or error proportion, of SS_y is denoted as $1\text{-}r^2$. Therefore:

$$SS_{error} = (1 - r^2) SS_y$$

$$\therefore r = \sqrt{1 + \frac{\sum_{n} (y_{data} - y_{model})^2}{\sum_{n} (y_{data} - \bar{y}_{data})^2}} \qquad \text{(F.3)}$$

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Annex G

Speed Decrement During Turning Circles

The following plots are speeds for a given rudder position during the turning circles. The first six graphs were data collected from the simulator and the following six graphs are sea trial data. The graphs are grouped in pairs (port and starboard) for each the three rudder positions 7, 15, and 30 degrees. Each graph shows the time response for three initial speed values, 9, 12, and 15 knots.

As the ship settles into a turning circle, the speed maintains a lower steady state value on average than the initial speed. The ratio between the average steady state speed and the initial speed is calculated and reported within each figure caption. These ratios are tabulated in Table 8. The ratio values are used to determine the non-linear gain as a function of rudder angle.

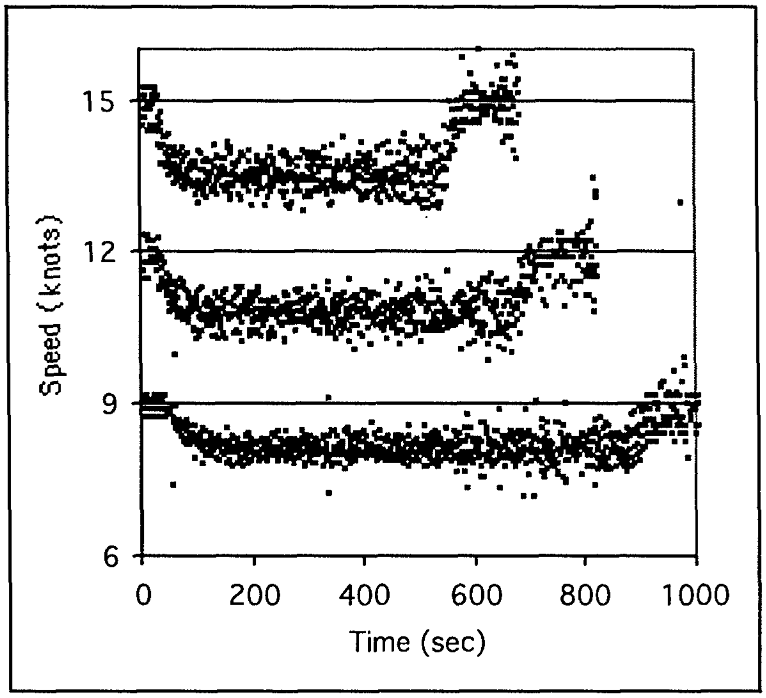

Figure G.1 *Simulation Port 7 (0.90)*

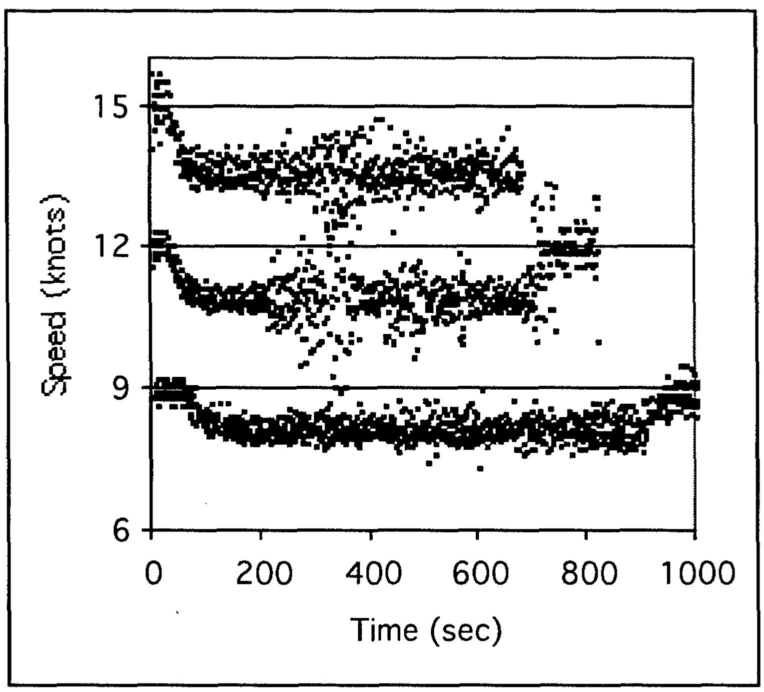

Figure G.2 *Simulation Starboard 7 (0.90)*

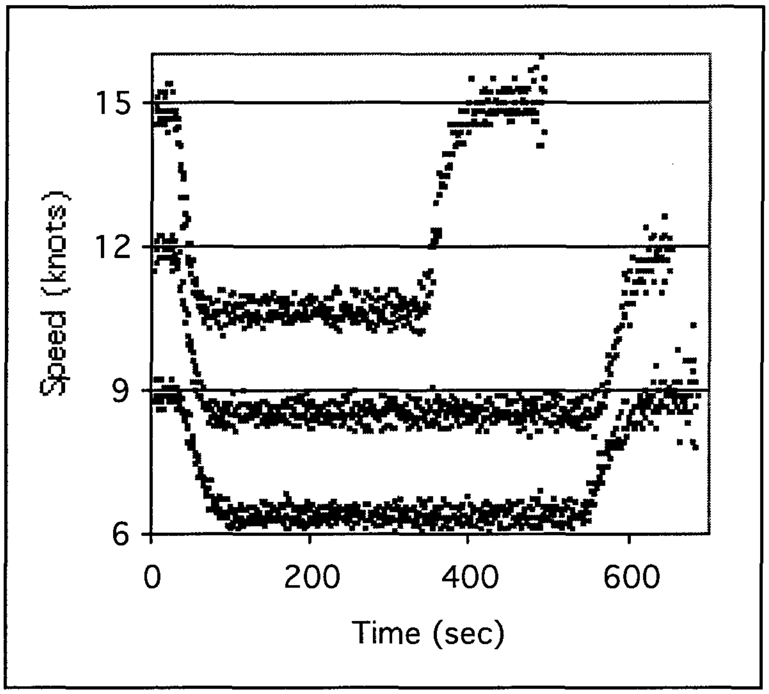

Figure G.3 Simulation Port 15 (0.71)

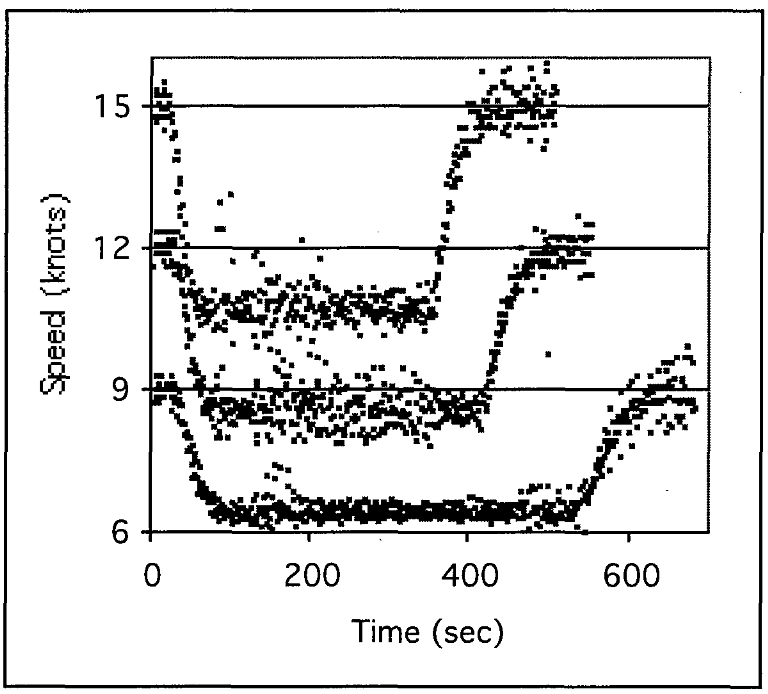

Figure G.4 Simulation Starboard 15 (0.71)

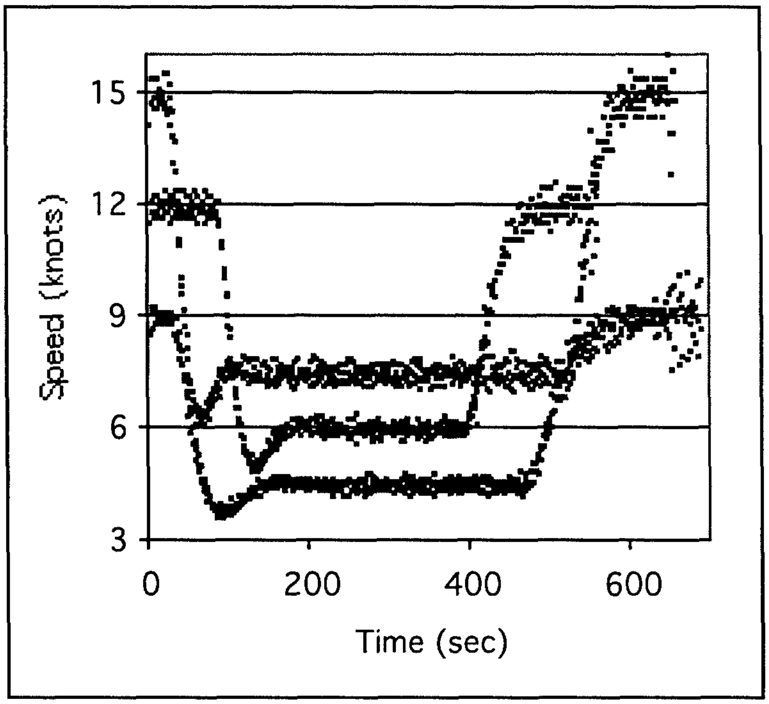

Figure G.5 Simulation Port 30 (0.49)

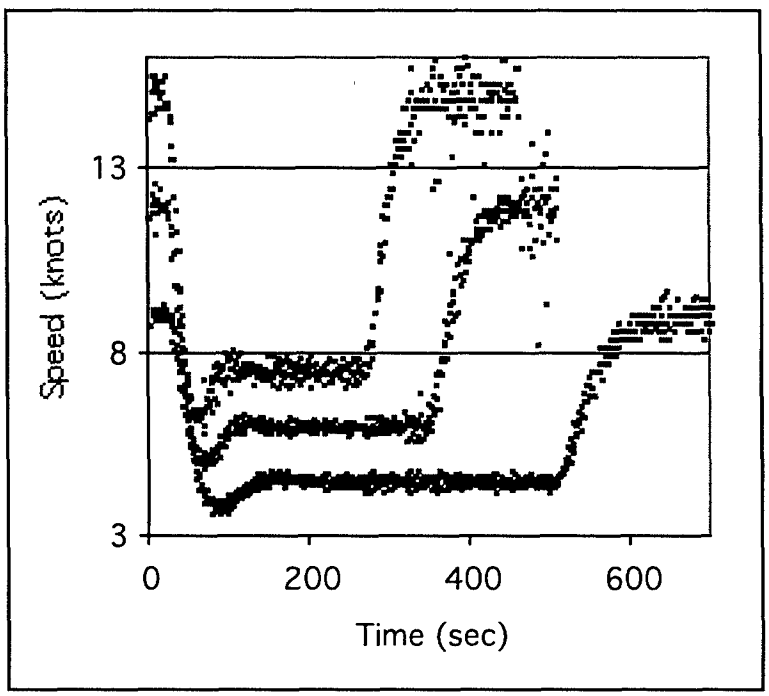

Figure G.6 Simulation Starboard 30 (0.50)

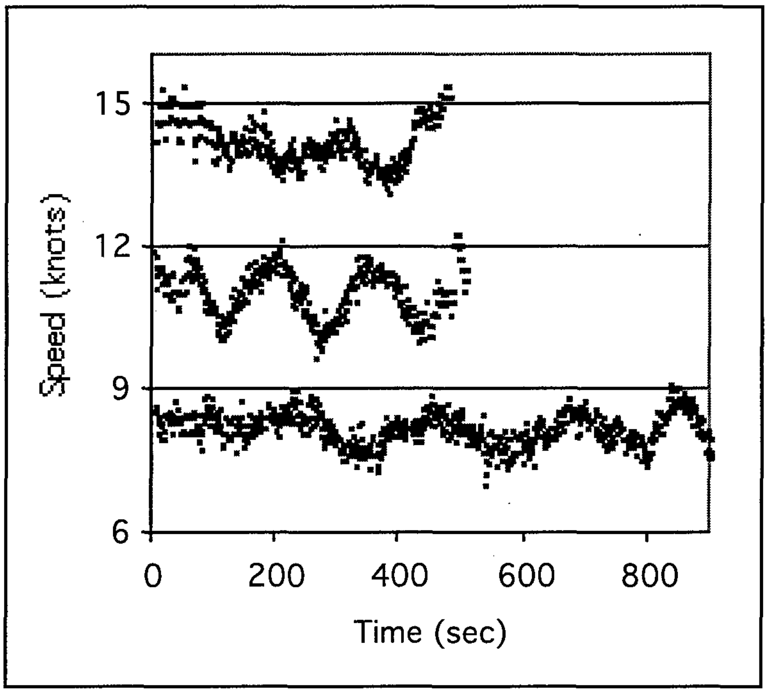

Figure G.7 Sea Trial Port 7 (0.91)

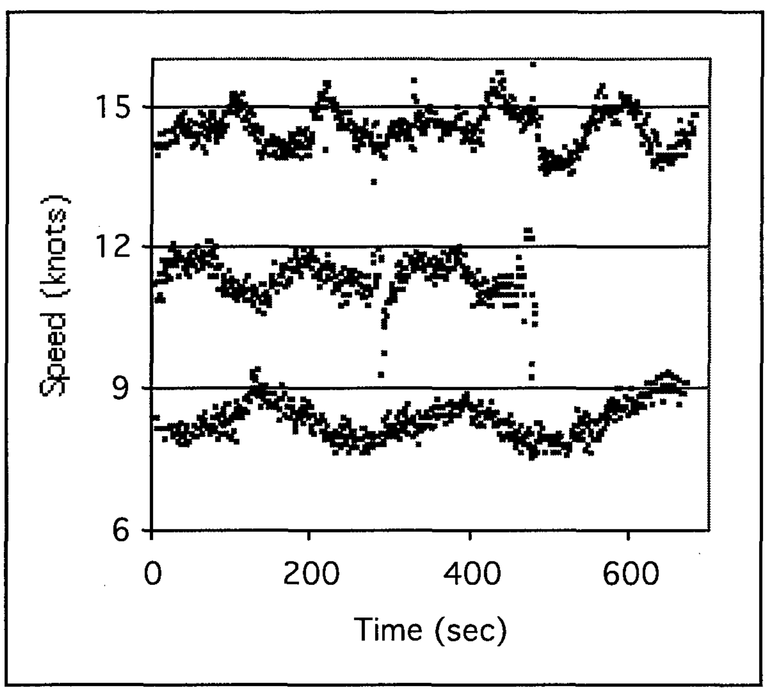

***Figure G.8** Sea Trial Starboad 7 (0.95)*

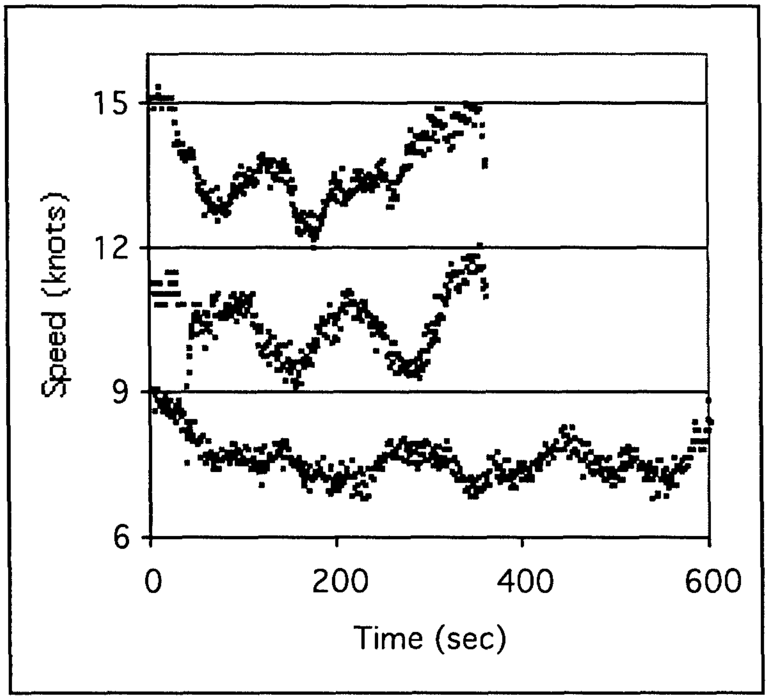

Figure G.9 Sea Trial Port 15 (0.84)

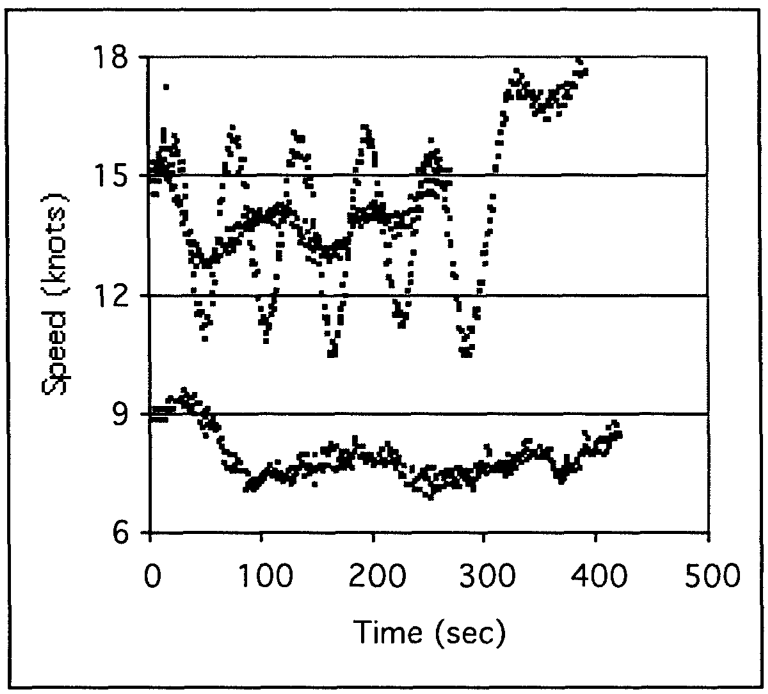

Figure G.10 Sea Trial Starboard 15 (0.89)

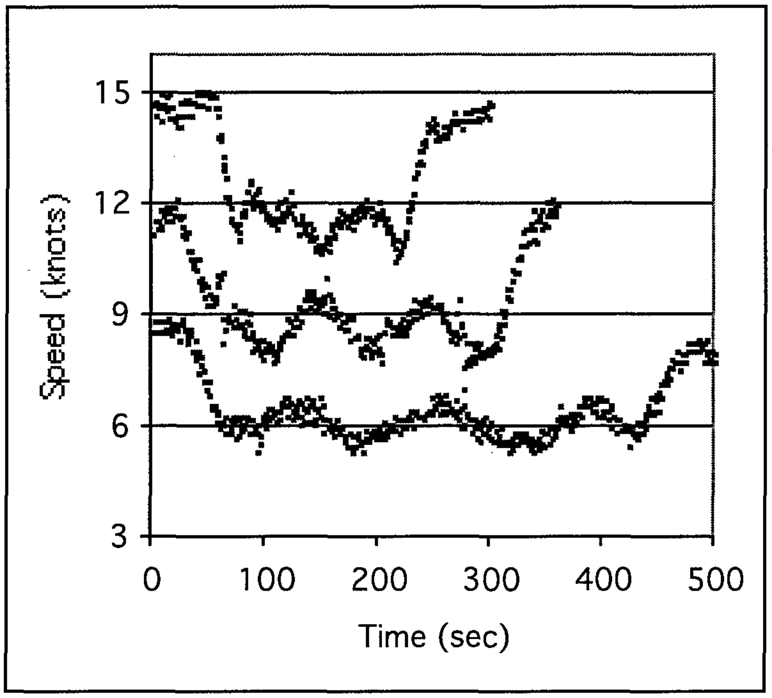

Figure G.11 Sea Trial Port 30 (0.72)

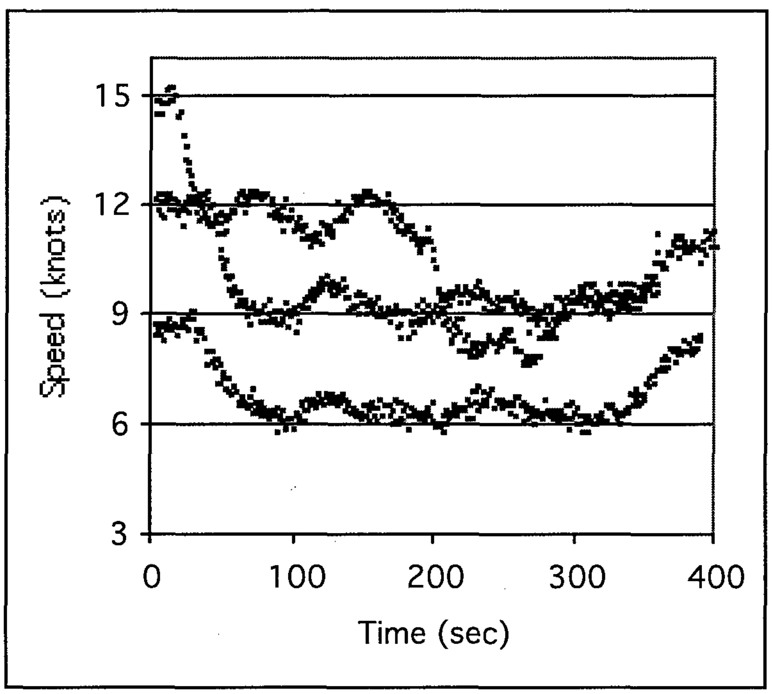

Figure G.12 Sea Trial Starboard 30 (0.75)

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List of symbols/abbreviations/acronyms/initialisms

CCT	Classical Control Theory
CF	Canadian Forces
DGPS	Differential Global Positioning System
kts	knots, nautical miles per hour
MARS	Maritime Surface/Subsurface
NNW	North North West
OOW	Officer of the Watch
PCT	Perceptual Control Theory
stbd	starboard
VRS	Virtual Reality Simulator
a	simple lag gain
a_n	normal acceleration
A	cross-sectional area for flow
b	simple lag time constant
c	system output; ideal lag gain and time constant
C_d	drag coefficient
C_{do}	drag coefficient when rudder angle is zero
d	distance travelled along a curvilinear path
d_o	distance travelled along a curvilinear path at time 0
D	drag force
D_o	drag force when rudder angle is zero
e	error signal
F	Force due to change of momentum

F_n	Force due to change of momentum, normal component
F_t	Force due to change of momentum, tangential component
G	plant transfer function
G1	transfer function between tangential velocity and its target velocity
G2	transfer function between angular velocity and its target velocity
G_c	controller transfer function
h	hull's extent into the water
H	feedback transfer function
k_1	Constant parameter that includes C_{do}, l, and h
k_2	Constant parameter that includes ρ, A, and m_{ship}
K1	Non-linear gain representing a drop in tangential velocity
K2	Non-linear gain representing a drop in angular velocity
l	ship's length
lat	latitude
lng	longitude
$L\{\ \}$	Laplace Transformation
m_{ship}	mass of ship
p_1	point one on spherical earth
p_2	point two on spherical earth
r	system reference; correlation; circular radius
R	spherical earth's radius
s	Laplace variable
t_1	time at point one
t_2	time at point two

u	speed command input
v	tangential (linear) velocity
$\dot{v}$	tangential (linear) acceleration
$\mathbf{v}_1$	flow speed vector before deflection
$\mathbf{v}_2$	flow speed vector after deflection
v_{ss}	steady state velocity
$v_{ss\alpha}$	steady state speed during a turning manoeuvre
v_t	target velocity
x	cartesian coordinate for ship's path
y	cartesian coordinate for ship's path
α	rudder command input
β	flow direction with respect to heading
θ	heading; arc that subtends two points on spherical earth
θ_o	initial heading
ρ	water density
τ	time constant
ω	angular velocity
ω_t	target velocity

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(U) The Maritime Surface and Subsurface (MARS) Virtual Reality Simulator (VRS) has a role in determining the transfer of training. Determining the transfer of training requires knowledge of the ship dynamics. This paper identifies the ship dynamics for a CF Bay-Class ship as well as a simulated ship. Simulation and sea trial data are collected and used for a system's identification exercise. Sources of error came from the sea state, crew behaviours, and the Differential Global Positioning System. Despite the sources of error, the data were relatively clean and the identification exercise was able to proceed. The analysis produced a piecewise linear and continuous model description for the ship's dynamics, with numerical coefficients. The advantage of the model is that it is computationally simpler than the full six degree of freedom model, and still captures over 90% of the experimental variance. The algorithms developed in this paper may be implemented to generate a real time ship dynamics model that could be downloaded into a portable MARS VRS system for onboard training and rehearsal.

15. KEYWORDS, DESCRIPTORS or IDENTIFIERS

(U)

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